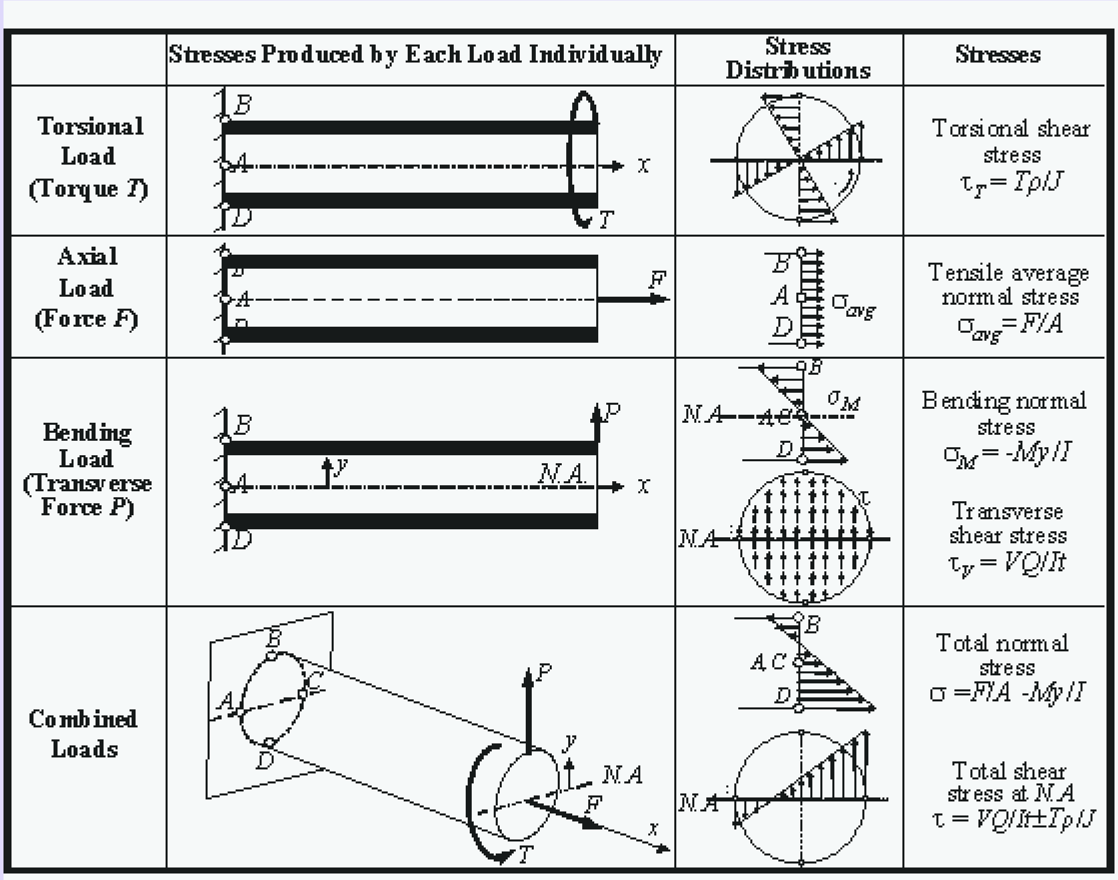
7.3 Design of members subjected to combined forces

7.3.1 General

In the previous chapters of Draft IS: 800 – LSM version, we have stipulated the codal provisions for determining the stress distribution in a member subjected to different types of stress resultants such as axial tensile force (Section 6), axial compressive force (Section 7) and bending moment along with transverse shear force (Section 8). Most often, the cross section of a member is subjected to several of these loadings *simultaneously*. As we shall see presently, we may combine the knowledge that we have acquired in the previous sections. As long as the relationship between stress and the loads is ***linear*** and the geometry of the member would ***not undergo significant change*** when the loads are applied, the principle of superposition can be applied. Here, as shown in Table 7.1, one typical case of combination due to tensile force *F*, torque *T* and transverse load *P* has been diagrammatically discussed.

In addition to the pure bending case, beams are often subjected to transverse loads which generate both bending moments M(x) and shear forces V(x) along the beam. The bending moments cause bending normal stresses s to arise through the depth of the beam, and the shear forces cause transverse shear-stress distribution through the beam cross section as shown in Fig. 7.3.

Table 7.1 Superposition of individual loads (a case study for solid circular shaft)



	Stresses Produced by Each Load Individually	Stress Distributions	Stresses
Torsional Load (Torque T)			Torsional shear stress $\tau_T = T\rho/J$
Axial Load (Force F)			Tensile average normal stress $\sigma_{avg} = F/A$
Bending Load (Transverse Force P)			Bending normal stress $\sigma_M = -My/I$ Transverse shear stress $\tau_V = VQ/It$
Combined Loads			Total normal stress $\sigma = F/A - My/I$ Total shear stress at N.A $\tau = VQ/It \pm T\rho/J$

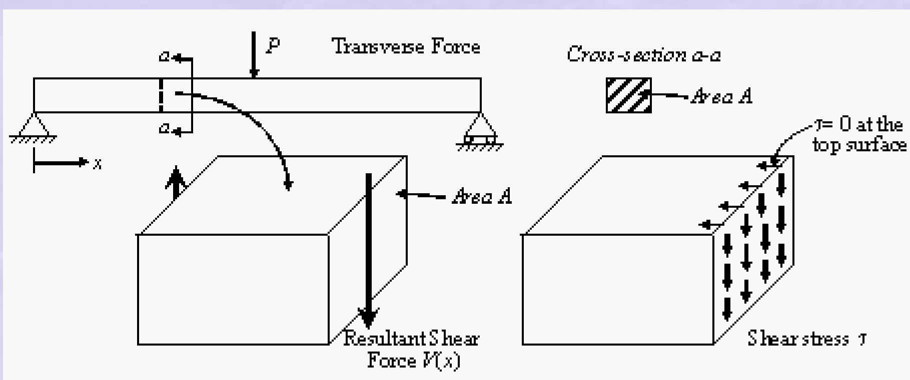


Fig.7.3 Beam with transverse shear force showing the transverse shear stress developed by it

7.3.2 General procedure for combined loading

- Identify the relevant equations for the problem and use the equations as a check list for the quantities that must be calculated.

- Calculate the relevant geometric properties (A, I_{yy} , I_{zz} , J) of the cross-section containing the points where stresses have to be found.

- At points where shear stress due to bending is to be found, draw a line perpendicular to the center-line through the point and calculate the first moments of the area (Q_y , Q_z) between free surface and the drawn line. Record the s-direction from the free surface towards the point where stress is being calculated.

- Make an imaginary cut through the cross-section and draw the free body diagram. On the free body diagram draw the internal forces and moments as per our sign conventions if subscripts are to be used in determining the direction of stress components. Using equilibrium equations to calculate the internal forces and moments.

- Using the equations identified, calculate the individual stress components due to each loading. Draw the torsional shear stress $\tau_{x\theta}$ and bending shear stress τ_{xs} on a stress cube using subscripts or by inspection. By examining the shear stresses in x, y, z coordinate system obtain τ_{xy} and τ_{xz} with proper sign.

- Superpose the stress components to obtain the total stress components at a point.

- Show the calculated stresses on a stress cube.

- Interpret the stresses shown on the stress cube in the x, y, z coordinate system before processing these stresses for the purpose of stress or strain transformation.

7.3.3 Design of member subjected to combined shear and bending:

In general, it has been observed that for structures, which are subjected to combined shear and bending the occurrence of high shear force is seldom. Here, high shear force has been designated as that shear force, which is more than 50 percent of the shear strength of the section. For structures where the factored value of applied shear force is less than or equal to 50 percent of the shear strength of the section no reduction in moment capacity of the section is required (refer clause 8.4 of section 8 of draft IS: 800 – LSM version) i.e. the moment capacity may be taken as M_d (refer clause 8.2 of section 8 and clause 9.2.1 of section 9 of draft IS: 800 – LSM version). If the factored value of actual shear force is more than 50 percent of the shear strength of the section, the section shall be checked and moment capacity, M_{dx}, shall be reduced depending upon classification of section (refer clause 9.2.2 of section 9 of draft IS: 800 – LSM version). This is done to take care of the increased resultant vector stress generated due to vector addition of stress due to high shear and bending moment at that particular section. The corresponding bending moment capacity is reduced by incorporating a factor 'β' $\beta = (2V/V_e - 1)^2$, which in turn depends upon the ratio of actual value of high shear and shear strength of that particular section. In no case, the maximum value of moment capacity shall exceed $1.2 Z_e f_y / \gamma_{m0}$, where, Z_e is elastic section modulus of the whole section γ_{m0}, is the partial safety factor against yield stress and buckling and f_y is the characteristic yield stress (250 N/mm^2). For semi-compact sections, this reduction in moment capacity is not required to be exercised as the section will be predominantly governed by the elastic moment capacity i.e. $Z_e f_y / \gamma_{m0}$, where the terms Z_e, γ_{m0} and f_y are as defined previously.

7.3.4 Design of member subjected to combined bending and axial force:

7.3.4.1 General

As with combined bending moment and axial force in the elastic range, the plane of zero strain moves from the centroid so that the ultimate stress distribution appears as follows:

- To compute the modified plastic moment, Mp', the stress blocks are divided into three components.

- The outer pair is equal and opposite stress blocks which provide the moment.

- The inner block acts in one direction, the area being balanced around the original neutral axis, defined by d_N. In the cases illustrated the inner block has a constant width, b or t_w, so that

$$M'_p = 2A_p\,\bar{y}\,\sigma_y;\;\; N = bd_N\,\sigma_y\,(\text{rectangle});\, N = t_w\,d_N\sigma_y\,(\text{I}-\text{section})$$

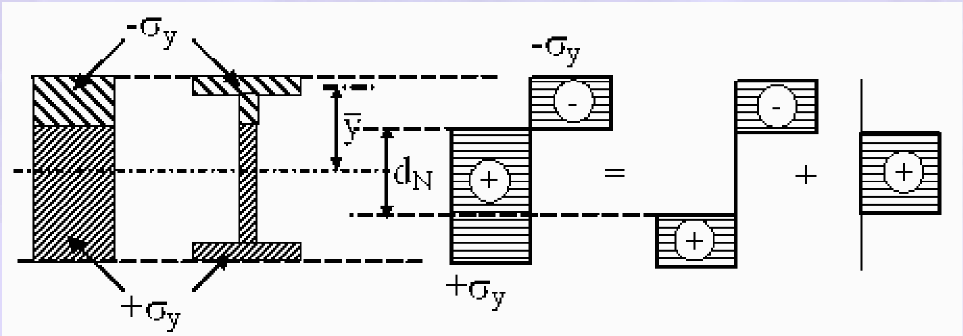


Fig.7.4 Stress block diagram for combined bending and axial force with reduced moment carrying capacity for I-section

We can draw interaction diagrams relating N and M for initial yield and ultimate capacity as follows:

When the web and the flange of an I-beam have different yield strengths it is possible to calculate the full plastic moment or the combined axial force and bending moment capacity taking into account the different yield strengths.

$$M_p = 2\sum_i A_i\, \bar{y}_i\, \sigma_{yi}$$

for all i segments with different yield stresses.

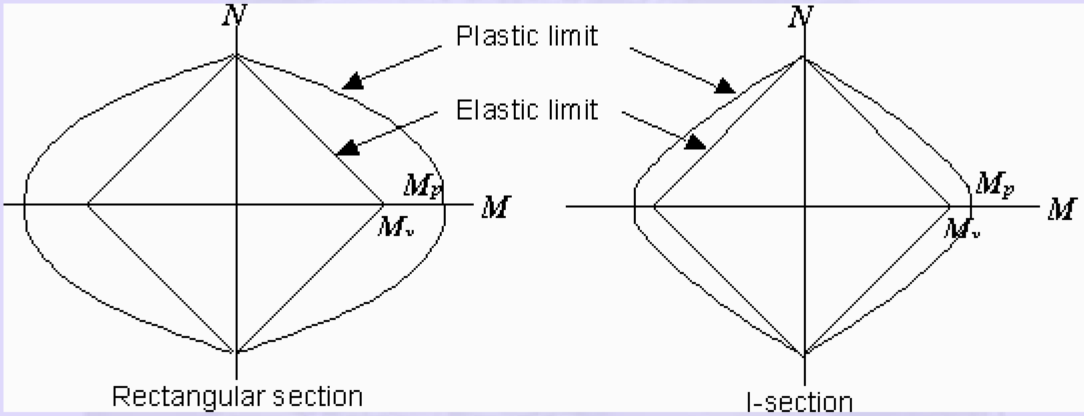


Fig.7.5 Interaction diagram of bending moment and axial tension or compression

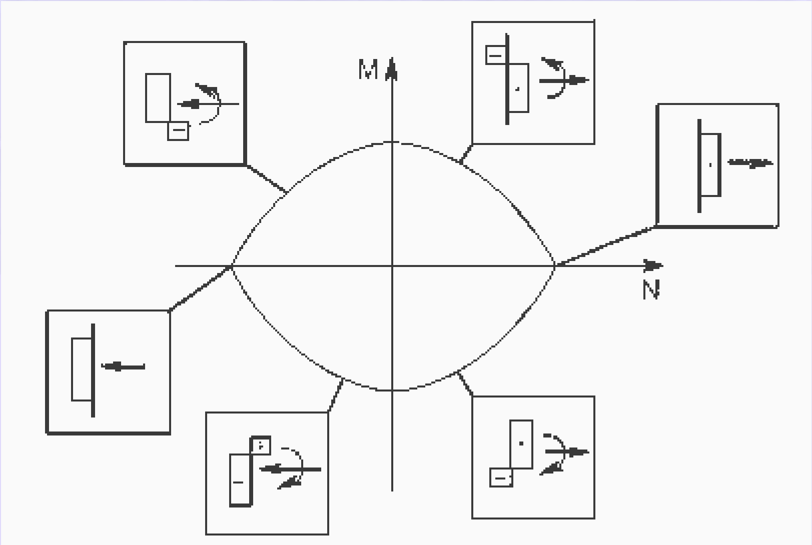


Fig.7.6 Plastic limit envelope with stress distributions for combined bending (M) and axial force (N)

7.3.4.2 Members subjected to combined bending & axial forces

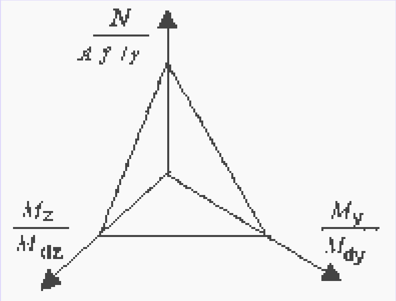


Fig. 7.7 Interaction surface of the ultimate strength under combined biaxial bending moment and axial tensile force

Any member subjected to bending moment and normal tension force should be checked for lateral torsional buckling and capacity to withstand the combined effects of axial load and moment at the points of greatest bending and axial loads. Figure 7.4 illustrates the type of three-dimensional interaction surface that controls the ultimate strength of steel members under combined biaxial bending and axial force. Each axis represents a single load component of normal force N, bending about the y and z axes of the section (M_y or M_z) and each plane corresponds to the interaction of two components.

7.3.4.2.1 Local capacity check (Section 9 Draft IS: 800 – LSM version)

For *Plastic and Compact sections*, the design of members subjected to combined axial load and bending moment shall satisfy the following interaction relationship:

$$\left(\frac{M_y}{M_{ndy}}\right)^{u1} + \left(\frac{M_z}{M_{ndz}}\right)^{u2} \leq 1.0$$

Where

M_y, M_z = factored applied moments about the minor and major axis of the cross section, respectively

M_{ndy}, M_{ndz} = design reduced flexural strength under combined axial force and the respective uniaxial moment acting alone, **(Cl. 9.3.1.2 of Draft IS: 800 – LSM version)**

N = factored applied axial force (Tension T, or Compression F)

N_d = design strength in tension (T_d) as obtained from **(section 6 Draft IS: 800 – LSM version)** or in compression and $N_d = A_g f_y / \gamma_{m0}$

A_g = gross area of the cross section

n = N / N_d

α_1, α_2 = constants as given in **Table 7.2** of new IS: 800 and shown below:

Table 7.2 Constants α_1 and α_2 *(Section 9.3.1.1of Draft IS: 800 – LSM version)*

Section	α_1	α_2
I and Channel	$5n \geq 1$	2
Circular tubes	2	2
Rectangular tubes	$1.66/(1-1.13n^2) \leq 6$	$1.66/(1-1.13n^2) \leq 6$
Solid rectangle	$1.73+1.8n^3$	$1.73+1.8n^3$

A typical interaction diagram for I beam-column segment with combined biaxial bending and compressive force is as shown in figure 7.8

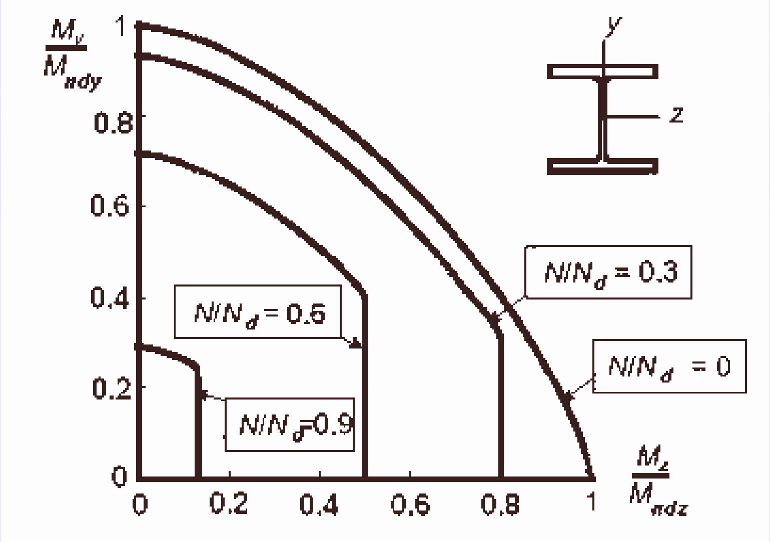


Fig. 7.8 Interaction curves for I beam-column segment in bi-axial bending and compression

The above interaction formulae, $\left(\frac{M_y}{M_{ndy}}\right)^{\alpha 1}+\left(\frac{M_z}{M_{ndz}}\right)^{\alpha 2}\le 1.0$ is a function of α_1 and α_2 and the values of α_1 and α_2 are in turn functions of the ratio $n = N / N_d$ (refer Table 7.2 above), where, N and N_d are factored applied axial force and design strength in tension or compression as defined earlier. It can be observed from the table 7.2 that for I-section or channel section, in case, the ratio $n = N / N_d$ is equal to 0.2, the value of α_1 becomes 1 and for $n = N / N_d > 0.2$, the value of α_1 is more than 1. As the value of α_1 increase above 1, the value of the component $\left(\frac{M_y}{M_{ndy}}\right)^{\alpha 1}$ gets reduced since the ratio $\left(\frac{M_y}{M_{ndy}}\right)^{\alpha 1}$ is always less than 1. The values of M_{ndy} and M_{ndz} are also proportionately reduced to accommodate the value of axial tension or compression depending upon type of sections. The ratio $n = N / N_d$ is also directly related in reducing the bending strength, M_{ndy} and M_{ndz}

The code stipulates **(as per clause 9.3.1.2 of Draft IS: 800 – LSM version)** that for plastic and compact sections without bolts holes, the following approximations may be used while calculating / deriving the values of design reduced flexural strength M_{ndz} and M_{ndy} under combined axial force and the respective uniaxial moment acting alone i.e. M_{ndz} and M_{ndy} acting alone. The values of reduced flexural strength of the section, (either M_{ndz} or M_{ndy}) is directly related to the geometry of a particular section. We will now discuss how these values are changing depending upon geometry of a particular section:

i) Plates (Section 9 of Draft IS: 800 – LSM version)

For rolled steel plates irrespective of their thickness the value of reduced flexural strength can be derived from the equation:– $M_{nd} = M_d\left(1-n_2\right)$. Here, the equation for reduced flexural strength is again a function of the ratio $n = N/N_d$, where, *N* and N_d are factored applied axial force and design strength in tension or compression as defined earlier. For smaller values of the ratio *n*, the reduction in flexural strength is not significant since the reduction in flexural strength is directly proportional to square of the ratio *n*. It is obvious from the equation that as the ratio tends towards the value 1, the amount of reduction in flexural strength increases and for extreme case, when the ratio is equal to 1, the value of reduced flexural strength is zero i.e. for this particular case no flexural or bending strength is available within the plate section. This situation also satisfies the Condition, $\frac{N}{N_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \leq 1.0$

ii) Welded I or H sections (Section 9 of Draft IS: 800 – LSM version)

For welded I or H sections, the reduced flexural strength about the major axis can be derived from the equation: – $M_{ndz} = M_{dz}(1-n)/(1-0.5a) \leq M_{dz}$ and about the minor axis: – $M_{ndy} = M_{dy}\left[1 - \left(\frac{n-a}{1-a}\right)^2\right] \leq M_{dy}$ where, $n = N/N_d$ and

$$a = (A - 2bt)/A/ \leq 0.5$$

Here the reduction in flexural strength for major axis is linearly and directly proportional to the ratio *n* and inversely proportional to the factor *a*, which is a reduction factor for cross sectional area ratio. It is pertinent to note that for a particular sectional area *A*, as the width and/or thickness of the flange of I or H section increases, the factor *a* reduces which in turn increases the value of M_{ndz}.

For minor axis, the reduction in flexural strength is non-linearly proportional to both the factors *n* and *a*, but as the value of the factor *n* increases considering other factor remaining unchanged, the value of M_{ndy} decreases, conversely as the value of the factor increases *a*, the value of M_{ndy} increases. For a particular case, when the numerical value of factor *n* is equal to 1 and the numerical value of the factor is *a* 0.5, the numerical value of M_{ndy} becomes zero. It can be observed that the factor *a* being the area ratio, it takes into account the effect of flange width and flange thickness. As the value of b or t_f increases, the value of the factor *a* reduces which in turn reduces further the value of design reduced flexural strength M_{ndy}.

iii) Standard I or H sections (Section 9 of Draft IS: 800 – LSM version)

For standard I or H sections, the reduced flexural strength about the major axis can be derived from the equation: – $M_{ndz} = 1.11 M_{dz}(1-n) \le M_{dz}$ and about the minor axis :–for $n \le 0.2$, $M_{ndz} = M_{ndy}$ and for, $n \le 0.2$ where, $M_{ndy} = 1.56 M_{dy}(1-n)(n+0.6)$

Unlike welded I or H sections, Here we do not find the factor *a*, but reduction in flexural strength for major axis is linearly and directly proportional to the ratio *n*. It is pertinent to note that for all cases, as the factor increases *n*, further reduction in reduced flexural strength of the member takes place. For a particular case, when the factor *n* becomes 1, the value of M_{ndz} reduces to zero.

For minor axis, no reduction in flexural strength takes place till the ratio $n = N/N_d$ is restricted to 0.2. When the value of *n* is more than 0.2, the reduction in flexural strength for minor axis is linearly and directly proportional to the ratio *n*. For a value of *n* = 1, the value of M_{ndy} reduces to zero.

iv) Rectangular Hollow sections and Welded Box sections (Section 9 of Draft IS: 800 – LSM version)

When the section is symmetric about both axis and without bolt holes, the reduced flexural strength about the major axis can be derived from the equation: –

$$M_{ndz} = M_{dz}(1-n)/(1-0.5a_w) \le M_{dz}$$

and about the minor axis:–

$$M_{ndy} = M_{dy}(1-n)/(1-0.5a_f) \le M_{dy}.$$

As indicated in above equations, for rectangular hollow sections and welded box sections, the reduction in flexural strength for both the axes, takes place in line with that of M_{ndz} for welded I or H sections as described earlier in ***ii)*** above. The only variation is, the factor *a* is replaced either by a_w for M_{ndz} or by a_f for M_{ndy}.

v) Circular Hollow Tubes without Bolt Holes (Section 9 of Draft IS: 800 – LSM version)

The reduced flexural strength about both the axes can be derived from the equation:– $M_{nd} = 1.04\ M_d \left(1 - n^{1.7}\right) \le M_d$. For smaller values of the ratio *n*, the reduction in flexural strength is not significant since the reduction in flexural strength is directly proportional to the power of 1.7 for the ratio *n*. When the ratio *n* is equal to 1, the value of reduced flexural strength is zero i.e. for this particular case no flexural or bending strength is available for the circular section.

Usually, the points of greatest bending and axial loads are either at the middle or ends of members under consideration. Hence, the member can also be checked, conservatively, as follows:

$$\frac{N}{N_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \le 1.0$$

where *N*, is the factored applied axial load in member under consideration, $N_d \left(A_g f_y / \gamma_{m0}\right)$ is the strength in tension as obtained from section **6,** M_z and M_y are the applied moment about the major and minor axes at critical region, M_{dz} and M_{dy} are the moment capacity about the major and minor axes in the absence of axial load i.e. when acting alone and A_g is the gross area of cross-section. This shows that in point of time, the summation of ratios of various components of axial forces and bending moments (including bi-axial bending moments) will cross the limiting value of 1.

For *Semi-compact section*s, when there is no high shear force **(as per 9.2.1 of Draft IS: 800 – LSM version)** semi-compact section design is satisfactory under combined axial force and bending, if the maximum longitudinal stress under combined axial force and bending f_x, satisfies the following criteria.

$$f_x \le f_y / \gamma_{m0}$$

For cross section without holes, the above criteria reduces to

$$\frac{N}{N_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \le 1.0$$

Where N_d, M_{dy}, M_{dz} are as defined earlier

7.3.4.2.2 Overall member strength check (section 9 of Draft IS : 800 - LSM version)

Members subjected to combined axial force and bending moment shall be checked for overall buckling failure considering the entire span of the member. This essentially takes care of lateral torsional buckling.

a) For Bending moment and Axial Tension, the member should be checked for lateral torsional buckling to satisfy overall stability of the member under reduced effective moment M_{eff} due to tension and bending. The reduced effective moment M_{eff}, can be calculated as per the equation $M_{eff} = \left[M - \psi T Z_{ec} / A\right] \le M_d$ but in no case shall exceed the bending strength due to lateral torsional buckling M_d **(as per 8.22 of Draft IS :800 - LSM version)**. Here M,T are factored applied moment and tension respectively, A is the area of cross section, Z_{ec} elastic section modulus of the section with respect to extreme compression fibre and the factor ψ is equal to 0.8 when tension and bending moments are varying independently or otherwise equal to 1. For extreme case, when the factor $\psi T Z_{ec} / A$ is equal to M, M_{eff} reduces to zero.

b) For Bending moment and Axial Compression , when the member is subjected to combined axial compression and biaxial bending, the section should be checked to satisfy the generalized interaction relationship as per the equation $\frac{P}{P_d} + \frac{K_y M_y}{M_{dy}} + \frac{K_z M_z}{M_{dz}} \le 1.0$. Here K_y, K_z are the moment amplification factor about minor and major axis respectively $\left(K_z = 1 - \frac{\mu_y P}{P_{dz}}\right)$ and $K_y = 1 - \frac{\mu_y P}{P_{dy}}$ where the factor μ_z and μ_y are dependent on equivalent uniform moment factor, b obtained from **Table 9.2 of Draft IS : 800 - LSM version**, according to the shape of the bending moment diagram between lateral bracing points in the appropriate plane of bending and non-dimensional slenderness ratio, l), P is the applied factored axial compression, M_y, M_z are the applied factor bending moments about minor and major axis of the member, respectively and P_d, M_{dy}, M_{dz} are the design strength under axial compression , bending about minor and major axis respectively , as governed by overall buckling criteria. The design compression strength, P_d , is the smallest of the minor axis (P_{dy}) and major axis (P_{dz}) buckling strength as obtained from **7.12 of Draft IS :800 - LSM version** and the design bending strength (M_{dz}) about major axis is equal to (M_d),where (M_d) is the design flexural strength about minor axis given by section **8.2.1 of Draft IS : 800 - LSM version**, when lateral torsional buckling is not significant and by section **8.2.2 of Draft IS: 800 - LSM version**, where lateral torsional buckling governs. For design Bending Strength about minor axis, $M_{dy} = M_d$ where, M_d is the design flexural strength about minor axis calculated using plastic section modulus for plastic and compact sections and elastic section modulus for semi-compact sections

c) The factors are as defined below.

μ_z is the larger of μ_{LT} and μ_{fz} as given below.

$$\mu_{LT} = 0.15\lambda_y\beta_{MLT} - 0.15 \le 0.90$$

$$\mu_{fz} = \lambda_z\left(2\beta_{Mz} - 4\right) + \left[\frac{Z_Z - Z_{eZ}}{Z_{eZ}}\right] \le 0.90$$

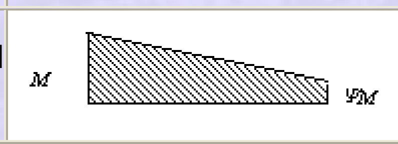
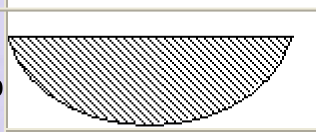
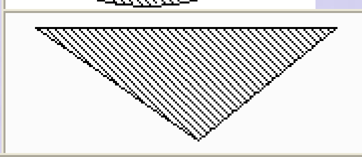
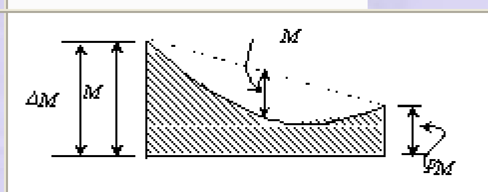
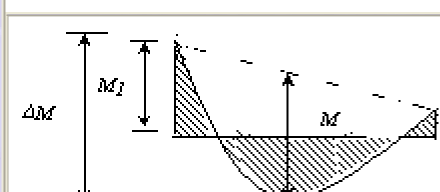
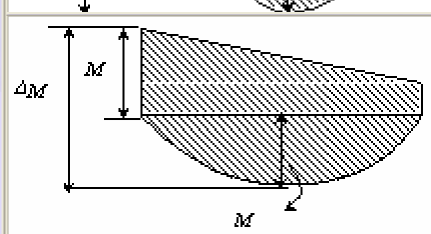
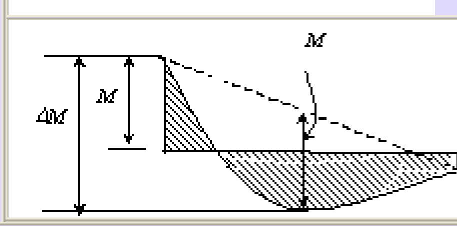
$$\mu_y = \lambda_y\left(2\beta_{My} - 4\right) + \left[\frac{Z_y - Z_{ey}}{Z_{ey}}\right] \le 0.90$$

$\beta_{My}, \beta_{Mz}, \beta_{MLT}$ = equivalent uniform moment factor obtained from **Table7.3 of Draft IS: 800-LSM version**, according to the shape of the bending moment diagram between lateral bracing points in the appropriate plane of bending

λ_y, λ_z = non-dimensional slenderness ratio **(7.1.2 of Draft IS 800- LSM version)** about the respective axis.

Table 7.3 OF Draft IS: 800-LSM Version, Equivalent uniform moment factor

(Section 9.3.2.2.1 of draft IS: 800-LSM version)

Particulars	BMD	b_m
Due to end moments		1.8-0.7y
Moment due to lateral loads		1.3
		1.4
moment due to lateral loads and end moments	   	1.8-0.7y+M_Q / Δ_M(0.74-0.5) $M_Q = \lvert M_{max} \rvert$ due to lateral load alone $\Delta_M = \lvert M_{max} \rvert$ (same curvature) $\Delta_M = \lvert M_{max} \rvert + \lvert M_{min} \rvert$ (reverse curvature)