

3.4 Analysis for lateral loads

3.4.1 Braced frames

In this section, simple hand methods for the analysis of statically determinate or certain low-redundant braced structures is reviewed.

Member Force Analysis

Analysis of the forces in a statically determinate triangulated braced frame can be made by the method of sections. For instance, consider a typical diagonal braced pin-jointed bay as shown in Fig. 3.10. When this bay is subjected to an external shear Q_i in i -th storey and external moments M_i and M_{i-1} at floors i and $i-1$, respectively, the force in the brace can be found by considering the horizontal equilibrium of the free body above section XX , thus,

$$F_{BC} \cos\theta = Q_i$$

Hence,

$$F_{BC} = Q_i / \cos\theta$$

The axial force F_{BD} in the column BD is found by considering moment equilibrium of the upper free body about C , thus

$$F_{BD} * \ell = M_{i-1}$$

Hence,

$$F_{BD} = M_{i-1} / \ell$$

Similarly the force F_{AC} in column AC is obtained from the moment equilibrium of the upper free body about B . It is given by

$$F_{AC} = M_i / \ell$$

This procedure can be repeated for the members in each storey of the frame. The member forces in more complex braced frames such as knee-braced, X-braced and K-braced frames can also be obtained by taking horizontal sections.

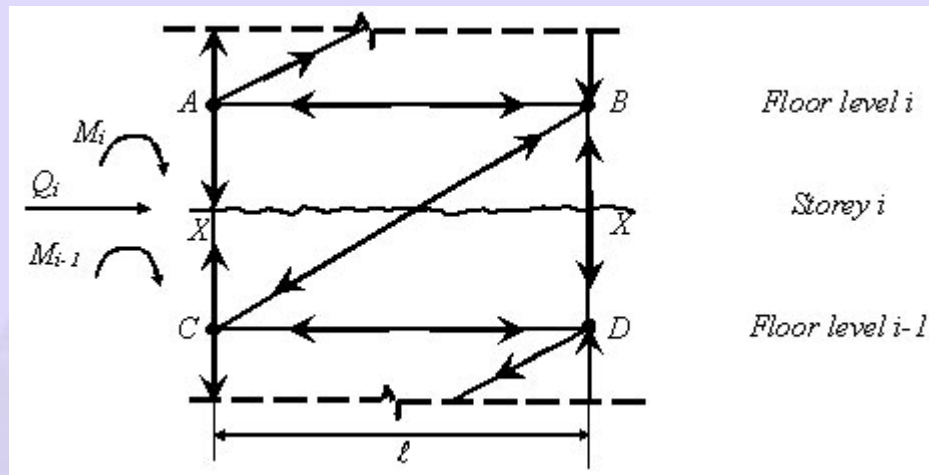


Fig. 3.10 Single diagonal braced panel

Drift Analysis

Drift in building frames is a result of flexural and shear mode contributions, due to the column axial deformations and to the diagonal and girder deformations, respectively. In low rise braced structures, the shear mode displacements are the most significant and, will largely determine the lateral stiffness of the structure. In medium to high rise structures, the higher axial forces and deformations in the columns, and the accumulation of their effects over a greater height, cause the flexural component of displacement to become dominant.

The storey drift in a braced frame reaches a maximum value at or close to the top of the structure and is strongly influenced by the flexural component of deflection. This is because the inclination of the structure caused by the flexural

component accumulates up the structure, while the storey shear component diminishes toward the top.

Hand analysis for drift allows the drift contributions of the individual frame members to be seen, thereby providing guidance as to which members should be increased in size to effectively reduce an excessive total drift or storey drift. The following section explains a method for hand evaluation of drift.

3.4.2 Moment-resisting frames

Multi-storey building frames subjected to lateral loads are statically indeterminate and exact analysis by hand calculation takes much time and effort. Using simplifying assumptions, approximate analyses of these frames yield good estimate of member forces in the frame, which can be used for checking the member sizes. The following methods can be employed for lateral load analysis of rigidly jointed frames.

- The Portal method.
- The Cantilever method
- The Factor method

The portal method and the cantilever method yield good results only when the height of a building is approximately more than five times its least lateral dimension. Either classical techniques such as slope deflection or moment distribution methods or computer methods using stiffness or flexibility matrices can be used if a more exact result is desired.

The portal method

This method is satisfactory for buildings up to 25 stories, hence is the most commonly used approximate method for analysing tall buildings. The following are the simplifying assumptions made in the portal method:

1. A point of contraflexure occurs at the centre of each beam.
2. A point of contraflexure occurs at the centre of each column.
3. The total horizontal shear at each storey is distributed between the columns of that storey in such a way that each interior column carries twice the shear carried by each exterior column.

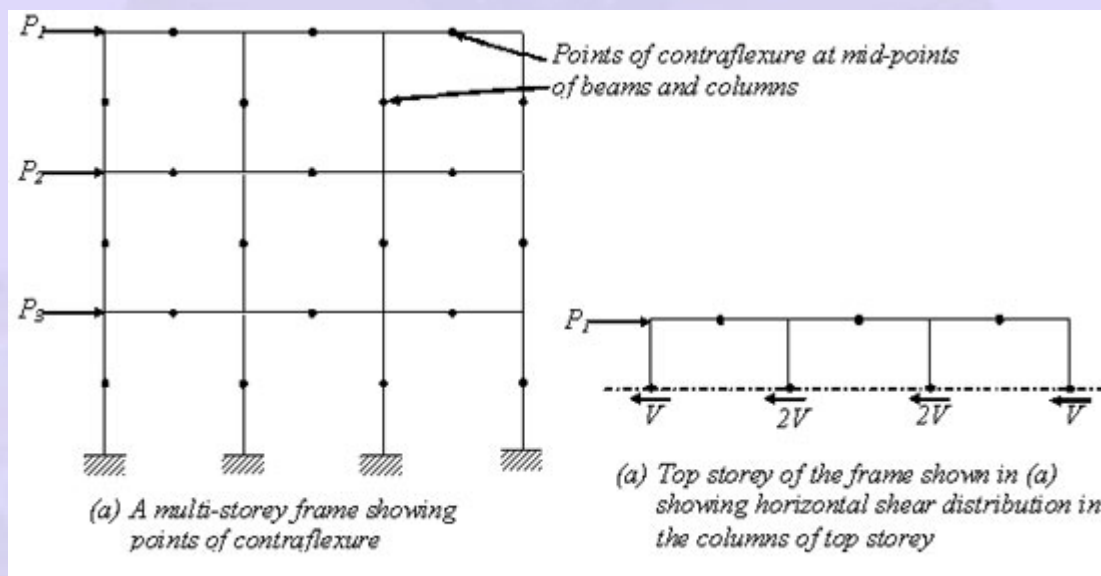


Fig.3.11 Portal method of analysis

The above assumptions convert the indeterminate multi-storey frame to a determinate structure. The steps involved in the analysis of the frame are detailed below:

1. The horizontal shears on each level are distributed between the columns of that floor according to assumption (3).

2. The moment in each column is equal to the column shear multiplied by half the column height according to assumption (2).

3. The girder moments are determined by applying moment equilibrium equation to the joints: by noting that the sum of the girder moments at any joint equals the sum of the column moments at that joint. These calculations are easily made by starting at the upper left joint and working joint by joint across to the right end.

4. The shear in each girder is equal to its moment divided by half the girder length. This is according to assumption (1).

5. Finally, the column axial forces are determined by summing up the beam shears and other axial forces at each joint. These calculations again are easily made by working from left to right and from the top floor down.

Assumptions of the Portal method of analysis are diagrammatically shown in Fig.3.11.

The cantilever method

This method gives good results for high-narrow buildings compared to those from the Portal method and it may be used satisfactorily for buildings of 25 to 35 storeys tall. It is not as popular as the portal method.

The simplifying assumptions made in the cantilever method are:

1. A point of contraflexure occurs at the centre of each beam
2. A point of contraflexure occurs at the centre of each column.
3. The axial force in each column of a storey is proportional to the horizontal distance of the column from the centre of gravity of all the columns of the storey under consideration.

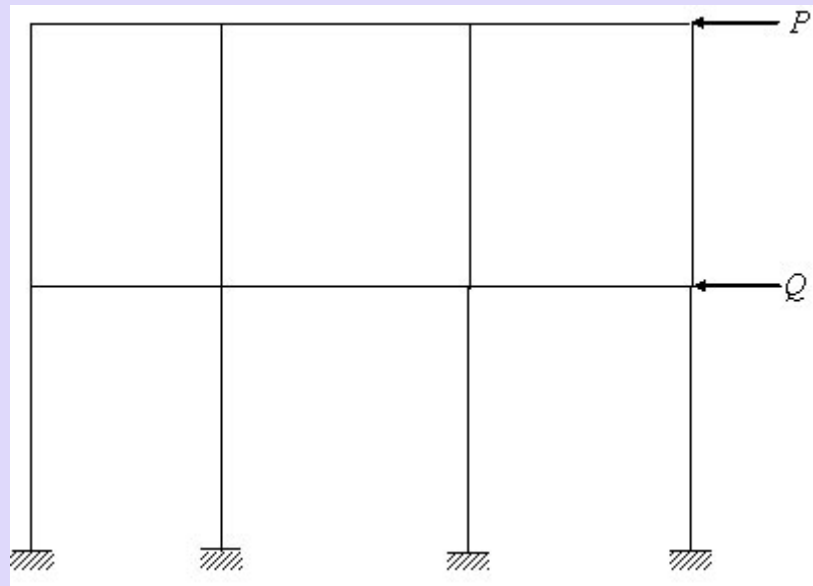


Fig. 3.12(a) Typical frame

The steps involved in the application of this method are:

1. The centre of gravity of columns is located by taking moment of areas of all the columns and dividing by sum of the areas of columns.

2. A lateral force P acting at the top storey of building frame is shown in Fig. 3.12(a). The axial forces in the columns are represented by F_1 , F_2 , F_3 and F_4 and the columns are at a distance of x_1 , x_2 , x_3 and x_4 from the centroidal axis respectively as shown in Fig. 3.12(b).

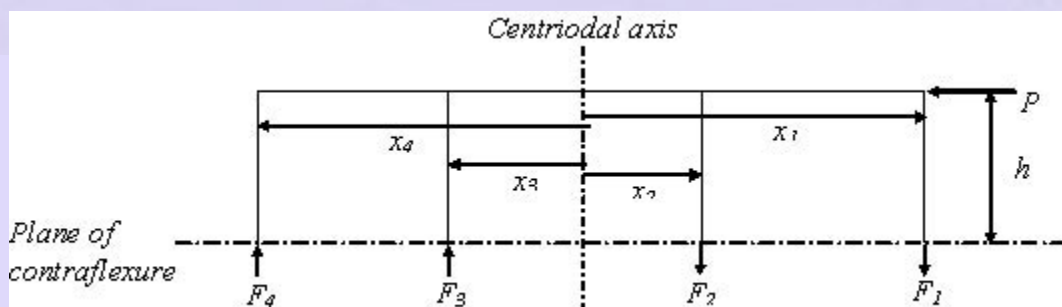


Fig. 3.12(b) Top storey of the frame above plane of contraflexure

By taking the moments about the centre of gravity of columns of the storey,

$$P h - F_1x_1 - F_2x_2 - F_3x_3 - F_4x_4 = 0$$

The axial force in one column may be assumed as F and the axial forces of remaining columns can be expressed in terms of F using assumption (3).

3. The beam shears are determined joint by joint from the column axial forces.
4. The beam moments are determined by multiplying the shear in the beam by half the span of beam according to assumption (1).
5. The column moments are found joint by joint from the beam moments.

The column shears are obtained by dividing the column moments by the half-column heights using assumption (2)

The factor method

The factor method is more accurate than either the portal method or the cantilever method. The portal method and cantilever method depend on assumed location of hinges and column shears whereas the factor method is based on assumptions regarding the elastic action of the structure. For the application of Factor method, the relative stiffness ($k = I/l$), for each beam and column should be known or assumed, where, I is the moment of inertia of cross section and l is the length of the member.

The application of the factor method involves the following steps:

1. The girder factor g , is determined for each joint from the following expression.

$$g = \frac{\sum k_c}{\sum k}$$

Where, $\sum k_c$ - Sum of relative stiffnesses of the column members meeting at that joint.

$\sum k$ - Sum of relative stiffnesses of all the members meeting at that joint.

Each value of girder factor is written at the near end of the girder meeting at the joint.

2. The column factor c , is found for each joint from the following expression

$$c = 1 - g$$

Each value of column factor c is written at the near end of each column meeting at the joint. The column factor for the column fixed at the base is one.

At each end of every member, there will be factors from step 1 or step 2. To these factors, half the values of those at the other end of the same member are added.

3. The sum obtained as per step 2 is multiplied by the relative stiffness of the respective members. This product is termed as column moment factor C , for the columns and the girder moment factor G , for girders.

4. Calculation of column end moments for a typical member ij - The column moment factors [C values] give approximate relative values of column end moments. The sum of column end moments is equal to horizontal shear of the

storey multiplied by storey height. Column end moments are evaluated by using the following equation,

$$M_{ij} = C_{ij} A$$

where, M_{ij} - moment at end i of the ij column

C_{ij} - column moment factor at end i of column ij

A - storey constant given by

$$A = \left(\frac{\text{Total horizontal Shear of Storey} \times \text{Height of the Storey}}{\text{Sum of the column end moment factors of the storey}} \right)$$

5. Calculation of beam end moments - The girder moment factors [G values] give the approximate relative beam end moments. The sum of beam end moments at a joint is equal to the sum of column end moments at that joint. Beam end moments can be worked out by using following equation,

$$M_{ij} = G_{ij} B$$

Where, M_{ij} - moment at end i of the ij beam

G_{ij} - girder moment factor at end i of beam ij

$$B = \left(\frac{\text{Sum of column moments at the joint}}{\text{Sum of the girder end moment factors of that joint}} \right)$$

B - joint constant given by

Illustration of calculation of G values:

Consider the joints B and C in the frame shown in Fig. 3.13.

$$\text{Joint B: } g_B = k_1 / (k_1 + k_2 + k_3)$$

$$c_B = 1 - g_B$$

$$\text{Joint C: } g_C = k_4 / (k_2 + k_4 + k_5)$$

$$c_C = 1 - g_C$$

As shown in Fig. 3.13, we should obtain values like x and y at each end of the beam and column. Thereafter we multiply them with respective k values to get the column or girder moment factors. Here, $G_{BC} = x k_2$ and $G_{CB} = y k_2$. Similarly we calculate all other moment factors.