4.4. Optimisation

Many design are possible to satisfy the functional requirements and a trial and error procedure may be employed to choose the optimal design. Selection of the best geometry of a structure or the member sizes are examples of optimal design procedures. The computer is best suited for finding the optimal solutions. Optimisation then becomes an automated design procedure, providing the optimal values for certain design quantities while considering the design criteria and constraints.

Computer-aided design involving user machine interaction and automated optimal design, characterised by pre-programmed logical decisions, based upon internally stored information, are not mutually exclusive, but complement each other. As the techniques of interactive computer-aided design develop, the need to employ standard routines for automated design of structural subsystems will become increasingly relevant.

The numerical methods of structural optimisation, with application of computers automatically generate a near optimal design in an interactive manner. A finite number of variables has to be established, together with the constraints relating to these variables. An initial guess-solution is used as the starting point for a systematic search for better designs and the process of search is terminated when certain criteria are satisfied.

Those quantities defining a structural system that are fixed during the automated design are called pre-assigned parameters or simply parameters and those quantities that are not pre-assigned are called design variables. The design variables cover the material properties, the topology of the structure, its geometry and the member sizes. The assignment of the parameters as well as the definition of their values are made by the designer, based on his experience.

Any set of values for the design variables constitutes a design of the structure. Some designs may be feasible while others are not. The restrictions that must be satisfied in in order to produce a feasible design are called constraints. There are two types of constraints: design constraints and behaviour constraints. Examples of design constraints are minimum thickness of a member, maximum height of a structure, etc. Limitations on the maximum stresses, displacement or buckling strength are typical examples of behaviour constraints. These constraints are expressed mathematically as a set of inequalities:

$$g_j(\{X\}) < 0 \qquad j = 1,2,....,m$$

Where {X} is the design vector, and m is the number of inequality constraints.

In addition, we have also to consider equality constraints of the form

$$h_j(\{X\}) = 0 \qquad j = 1,2,....,k$$

Where k is the number of equality constraints.

Example

The three bar truss example first solved by Schmitt is shown in Fig 4.6. The applied loadings and the displacement directions are also shown in this figure.

1. Design constraints: The conditions that the area of members cannot be less than zero can be expressed as

$$g_1 \equiv -X_1 \leq 0$$
$$g_2 \equiv -X_2 \leq 0$$

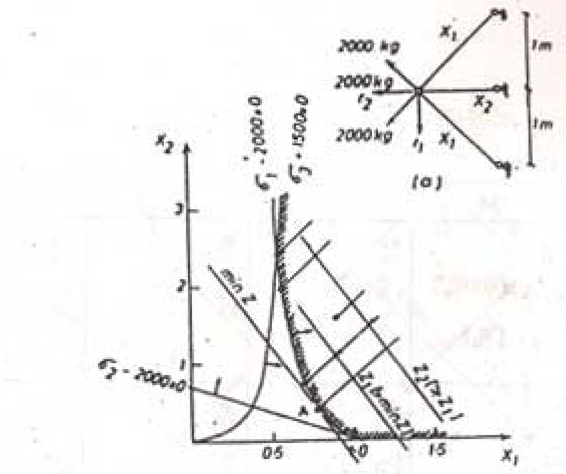
2. Behaviour constriants: The three members of the truss should be safe, that is the stresses in them should be less than the allowable stresses in tension (2,000kg/cm^2) and compression (1,500kg/cm^2). This is expressed as

$$g_3 \equiv \sigma_1 - 2000 \leq 0 \text{ Tensile stress limit in member 1}$$
$$g_4 \equiv -\sigma_1 - 1500 \leq 0 \text{ compressive stress limitaion in member 2 and so on}$$
$$g_5 \equiv \sigma_2 - 2500 \leq 0$$
$$g_6 \equiv -\sigma_2 - 1500 \leq 0$$
$$g_7 \equiv \sigma_3 - 2000 \leq 0$$
$$g_8 \equiv -\sigma_3 - 2000 \leq 0$$

3. Stress-force relationships: Using the stress-strain relationship s = [E]{Δ} and the force-displacement relationship F=[K]{Δ}, the stress-force relationship is obtained as {σ}=[E][K]$^{-1}${F}which can be shown as

$$\sigma_1 = 2000\left(\frac{X_2 + \sqrt{2}X_1}{2X_1X_2 + \sqrt{2}X_1^{\,2}}\right)$$
$$\sigma_2 = 2000\left(\frac{\sqrt{2}X_1}{2X_1X_2 + \sqrt{2}X_1^{\,2}}\right)$$
$$\sigma_3 = 2000\left(\frac{X_2}{2X_1X_2 + \sqrt{2}X_1^{\,2}}\right)$$

4. Constraint design inequalities: Only constraints g_3, g_5, g_8 will affect the design. Since these constraints can now be expressed interms of design variable X_1 and X_2 using the stress-force relationship derived above, they can be represented as the area on one side of the straight line in the two-dimensional plot. (Fig 4.6)



Example of a 3 bar truss.

Fig 4.6