4.7. Mathematical programming

It is difficult to anticipate which of the constraints will be critical at the optimum. Therefore, the use of inequality constraints is essential for a proper formulation of the optimal design problem.

The mathematical programming (MP) methods are included to solve the general optimisation problem by numerical search algorithms while being general regarding the objective function and constraints. On the other hand, approximations are often required tube efficient on large practical problems such as space structures.

Optimal design processes involves the minimization of weight subject to certain constraints. Mathematical programming methods and structural theorems are available to achieve such a design goal.

Of the various mathematical programming methods available for optimisation, the linear programming methods is widely adopted in structural engineering practice because of its simplicity. The objective function, which is the minimisation of weight, is linear and a set of constraints, which can be expressed by linear equations involving the unknowns (area, moment of inertia, etc., of the members), are used for solving the problems. This can be mathematically expressed as follows:

Suppose it is required to find a specified number of design variables x_1, $x_2 ... x_n$ such that the objective function.

$$Z = C_1x_1 + C_2x_2 + ... C_nx_n$$

is minimised, satisfying the constraints.

$$a_{11}x_1 = a_{12}x_2 + ...a_{1n}x_n \le b_1$$
$$a_{21}x_1 = a_{22}x_2 + ...a_{2n}x_n \le b_2$$
$$.$$
$$.$$
$$a_{m1}x_1 = a_{m2}x_2 + ...a_{mn}x_n \le b_m$$

The simplex algorithm is a versatile procedure for solving linear programming (LP) problems with a large number of variables and constraints.

The simplex algorithm is now available in the form of standard computer software package which uses the matrix representation of the variables and constraints, especially when their number is very large.

The above set of equations is expressed in the matrix form as follows:

Find $X = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix}$ Which minimised

The objective function $f(x) = \sum_{n-1}^{n} C_i x_i$

subject to the constraints

$$\sum_{k-1}^{n} a_{jk} x_k \le b_j ,\ j = 1, 2, ...m$$
$$\text{and } x_i \ge 0,\ i = 1, 2, ...n$$

Where C_i, a_{jk} and b_j are constants

The stiffness method of analysis is adopted and the optimisation is achieved by mathematical programming

The structure is divided into a number of groups and the analysis is carried out groupwise. Then the member forces are determined. The critical members are found out from each group. From the initial design, the objective function and the constraints are framed. Then, by adopting the fully stressed design (optimality criteria) method, the linear programming problem is solved and the optimal solution found out. In each group every member is designed for the fully stressed condition and the maximum size required is assigned for all the members in that group. After completion of the design, one more analysis and design routine for the structure as a whole is completed for alternative cross-section.