

## 9 Voltage Regulation

Modern power systems operate at some standard voltages. The equipments working on these systems are therefore given input voltages at these standard values, within certain agreed tolerance limits. In many applications this voltage itself may not be good enough for obtaining the best operating condition for the loads. A transformer is interposed in between the load and the supply terminals in such cases. There are additional drops inside the transformer due to the load currents. While input voltage is the responsibility of the supply provider, the voltage at the load is the one which the user has to worry about. If undue voltage drop is permitted to occur inside the transformer the load voltage becomes too low and affects its performance. It is therefore necessary to quantify the drop that takes place inside a transformer when certain load current, at any power factor, is drawn from its output leads. This drop is termed as the voltage regulation and is expressed as a ratio of the terminal voltage (the absolute value per se is not too important).

The voltage regulation can be defined in two ways - Regulation Down and Regulation up. These two definitions differ only in the reference voltage as can be seen below.

Regulation down: This is defined as " the change in terminal voltage when a load current at any power factor is applied, expressed as a fraction of the no-load terminal voltage".

Expressed in symbolic form we have,

$$Regulation = \frac{|V_{nl}| - |V_l|}{|V_{nl}|} \quad (52)$$

$V_{nl}$  and  $V_l$  are no-load and load terminal voltages. This is the definition normally used in the case of the transformers, the no-load voltage being the one given by the power

supply provider on which the user has no say. Hence no-load voltage is taken as the reference.

Regulation up: Here again the regulation is expressed as the ratio of the change in the terminal voltage when a load at a given power factor is thrown off, and the on load voltage. This definition if expressed in symbolic form results in

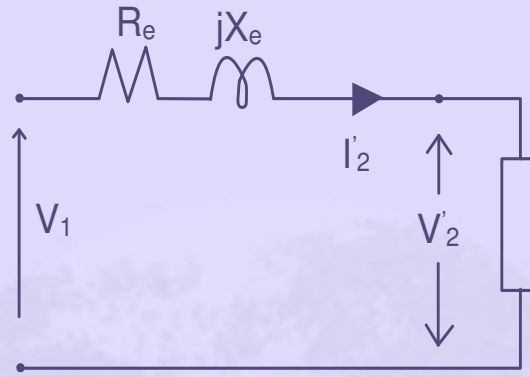
$$\text{Regulation} = \frac{|V_{nl}| - |V_l|}{|V_l|} \quad (53)$$

$V_{nl}$  is the no-load terminal voltage.

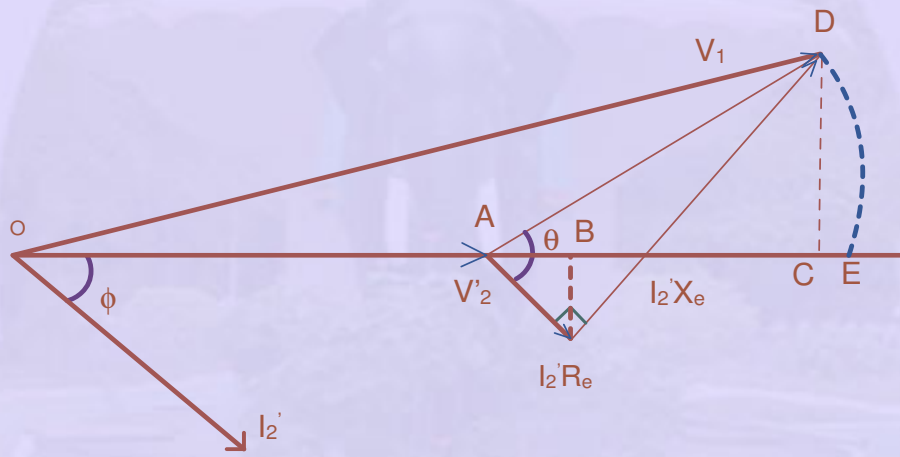
$V_l$  is load voltage. Normally full load regulation is of interest as the part load regulation is going to be lower.

This definition is more commonly used in the case of alternators and power systems as the user-end voltage is guaranteed by the power supply provider. He has to generate proper no-load voltage at the generating station to provide the user the voltage he has asked for. In the expressions for the regulation, only the numerical differences of the voltages are taken and not vector differences.

In the case of transformers both definitions result in more or less the same value for the regulation as the transformer impedance is very low and the power factor of operation is quite high. The power factor of the load is defined with respect to the terminal voltage on load. Hence a convenient starting point is the load voltage. Also the full load output voltage is taken from the name plate. Hence regulation up has some advantage when it comes to its application. Fig. 23 shows the phasor diagram of operation of the transformer under loaded condition. The no-load current  $I_0$  is neglected in view of the large magnitude of  $I_2'$ . Then



(a) Equivalent Circuit



(b) Phasor Diagram

Figure 23: Regulation of Transformer

$$I_1 = I_2'$$

$$V_1 = I_2'(R_e + jX_e) + V_2' \quad (54)$$

$$\begin{aligned} OD &= V_1 = \sqrt{[OA + AB + BC]^2 + [CD]^2} \\ &= \sqrt{[V_2' + I_2'R_e \cos \phi + I_2'X_e \sin \phi]^2 + [I_2'X_e \cos \phi - I_2'R_e \sin \phi]^2} \end{aligned} \quad (55)$$

$\phi$  - power factor angle,

$\theta$  - internal impedance angle =  $\tan^{-1} \frac{X_e}{R_e}$

Also,

$$V_1 = V_2' + I_2' \cdot (R_e + jX_e) \quad (56)$$

$$= V_2' + I_2'(\cos \phi - j \sin \phi)(R_e + jX_e)$$

$$\therefore \text{Regulation } R = \frac{|V_1| - |V_2'|}{|V_2'|} = \sqrt{(1 + v_1)^2 + v_2'^2} - 1 \quad (57)$$

$$(1 + v_1)^2 + v_2'^2 \simeq (1 + v_1)^2 + v_2'^2 \cdot \frac{2(1 + v_1)}{2(1 + v_1)} + \left[\frac{v_2'^2}{2(1 + v_1)}\right]^2 = \left(1 + v_1 + \frac{v_2'^2}{2(1 + v_1)}\right)^2 \quad (58)$$

$$\text{Taking the square root} \quad (59)$$

$$\sqrt{(1 + v_1)^2 + v_2'^2} = 1 + v_1 + \frac{v_2'^2}{2(1 + v_1)} \quad (60)$$

where  $v_1 = e_r \cos \phi + e_x \sin \phi$  and  $v_2 = e_x \cos \phi - e_r \sin \phi$

$e_r = \frac{I_2'R_e}{V_2'}$  = per unit resistance drop

$e_x = \frac{I_2'X_e}{V_2'}$  = per unit reactance drop

as  $v_1$  and  $v_2$  are small.

$$\therefore R \simeq 1 + v_1 + \frac{v_2'^2}{2(1 + v_1)} - 1 \simeq v_1 + \frac{v_2'^2}{2} \quad (61)$$

$$\therefore \text{regulation } R = e_r \cos \phi \pm e_x \sin \phi + \frac{(e_x \sin \phi - e_r \cos \phi)^2}{2} \quad (62)$$

$$\frac{v_2^2}{2(1+v_1)} \simeq \frac{v_2^2}{2} \cdot \frac{(1-v_1)}{(1-v_1^2)} \simeq \frac{v_2^2}{2} \cdot (1-v_1) \simeq \frac{v_2^2}{2} \quad (63)$$

Powers higher than 2 for  $v_1$  and  $v_2$  are negligible as  $v_1$  and  $v_2$  are already small. As  $v_2$  is small its second power may be neglected as a further approximation and the expression for the regulation of the transform boils down to

$$\text{regulation } R = e_r \cos \phi \pm e_x \sin \phi$$

The negative sign is applicable when the power factor is leading. It can be seen from the above expression, the full load regulation becomes zero when the power factor is leading and  $e_r \cos \phi = e_x \sin \phi$  or  $\tan \phi = e_r/e_x$  or the power factor angle  $\phi = \tan^{-1}(e_r/e_x) = \tan^{-1}(R_e/X_e)$  leading.

Similarly, the value of the regulation is maximum at a power factor angle  $\phi = \tan^{-1}(e_x/e_r) = \tan^{-1}(X_e/R_e)$  lagging.

An alternative expression for the regulation of a transformer can be derived by the method shown in Fig. 24. Here the phasor are resolved along the current axis and normal to it.

We have,

$$OD^2 = (OA + AB)^2 + (BC + CD)^2 \quad (64)$$

$$= (V_2' \cos \phi + I_2' R_e)^2 + (V_2' \sin \phi + I_2' X_e)^2 \quad (65)$$

$$\therefore \text{Regulation } R = \frac{OD - V_2'}{V_2'} = \frac{OD}{V_2'} - 1 \quad (66)$$

$$\sqrt{\frac{(V_2' \cos \phi + I_2' R_e)^2}{V_2'^2} + \frac{(V_2' \sin \phi + I_2' X_e)^2}{V_2'^2}} - 1 \quad (67)$$

$$= \sqrt{(\cos \phi + R_{p.u})^2 + (\sin \phi + X_{p.u}^2)} - 1 \quad (68)$$



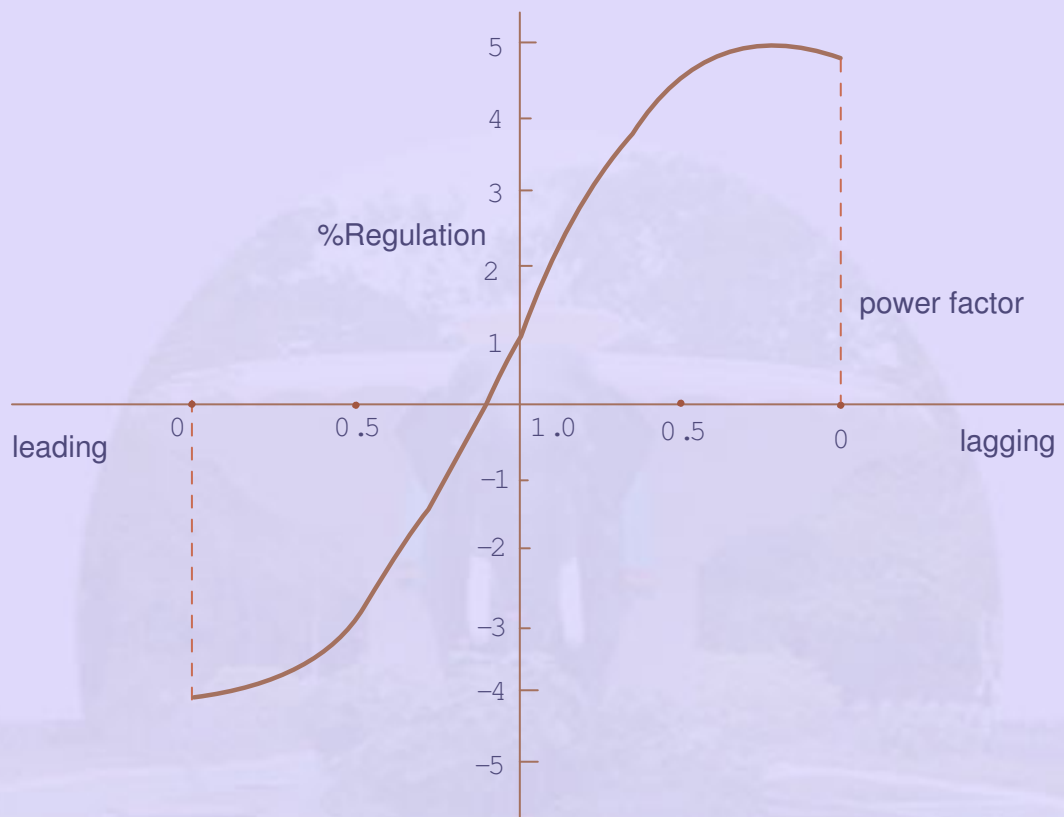


Figure 25: Variation of Full Load Regulation with Power Factor

A low value of the short circuit impedance /reactance results in a large short circuit current in case of a short circuit. This in turn results in large mechanical forces on the winding. So, in large transformers the short circuit impedance is made high to give better short circuit protection to the transformer which results in poorer regulation performance. In the case of transformers provided with taps on windings, so that the turns ratio can be changed, the voltage regulation is not a serious issue. In other cases care has to be exercised in the selection of the short circuit impedance as it affects the voltage regulation.

