10.1 Characteristics of critical flow

The characteristics of critical flow are

(i) The specific energy and specific force are minimum for the given discharge.

(ii) The Froude number is equal to unity.

(iii) For a given specific energy the discharge is maximum at the critical flow.

(iv) The velocity head is equal to half the hydraulic depth in a channel of small slope.

(v) The velocity of flow in a channel of small slope with uniform velocity distribution, is equal to the celerity of small gravity waves $\left(C = \sqrt{gh}\right)$ is shallow water caused by local disturbance.

(vi) Flow at the critical state is unstable.

Critical flow may occur at a particular section or in the entire channel, then the flow in the channel is called "Critical flow".

$y_c = f\left(A, D\right)$ for a given discharge.

For a prismatic channel for a given discharge the critical depth is constant at all sections of a channel. The bed slope which sustains a given discharge at a uniform and critical depth is called "Critical slope S_c". A channel slope causing slower flow in sub critical state for a given discharge is called "sub critical slope or mild slope". A slope greater than the critical slope is called steep slope or super critical slope.

10.1.1 Critical Flow

For a given specific energy and discharge per unit width q, there are two possible (real) depths of flow, and that transition from one depth to the other can be accomplished under certain situations. These two depths represented on the two different limbs of the E-y curve separated by the crest c, are characteristic of two different kinds of flow; a rational way to understand the nature of the difference between them is to consider first the flow represented by the point c. Here the flow is in a critical condition, poised between two alternative flow regimes, and indeed the word “critical " is used to describe this state of flow; it may be defined as the state at which the specific energy E is a minimum for a given q.

10.1.2 Analytical Properties of Critical Flow

Consider the Specific energy equation $E = y + \frac{v^2}{2g} = y + \frac{q^2}{2gy^2}$

in which y is the depth of flow and q is the discharge per unit width.

Differentiating the above equation with respect to y and equating to zero it can be written as

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0$$

$$\therefore\ q^2 = gy_c^3 \quad \text{or} \quad y_c = \sqrt[3]{\frac{q^2}{g}}$$

$$\text{and} \quad V_c^2 = gy_c$$

The subscript c indicates critical flow conditions.

Thus the critical depth y_c is a function of discharge per unit width alone.

Further, the above equation it can be written as

$$\frac{V_c^2}{2g} = \frac{1}{2} y_c \ .$$

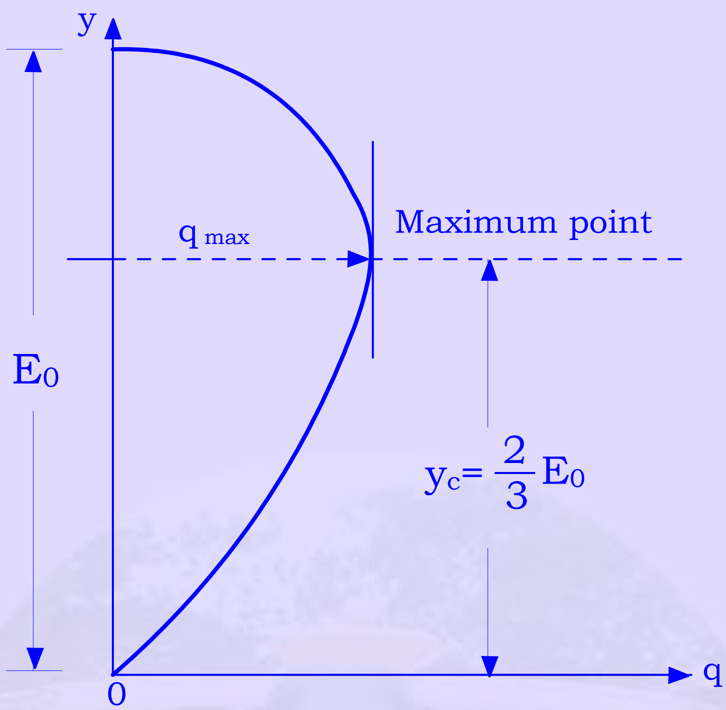
Thus the specific energy for critical flow can be expressed as

$$E_c = y_c + \frac{V_c^2}{2g} = \frac{3}{2} y_c$$

$$\text{or} \quad y_c = \frac{2}{3} E_c$$

The second derivative should be negative i.e., $\frac{d^2E}{dy^2} = -\text{ve}$

The above equations are established by considering the variation of specific energy with y for a given q. Clearly the curve will be of the general form as shown in Figure.



Variation of the Discharge with depth for a given specific energy value

How q varies with y for a given E= E_o?

When $y \to E_o$ and then $q \to 0$. *Similarly*, *when* $y \to 0,\ q \to 0$, and there will clearly be a maximum value of q for some value of y between 0 and E_0 (y cannot be greater than E_0). The relationship can be written as $q^2 = 2gy^2(E_o - y)$ and differentiating the above equation with respect to y,

$$2q\frac{dq}{dy} = 4gyE_o - 6gy^2 = 0$$

$$\therefore\ 6gy_c^2 = 4gyE_0$$

$$\text{i.e.,}\ y_c = \frac{2}{3}E_0$$

Differentiating again it can be established that $\frac{d^2E}{dy^2} = +ve$

Alternative approach:

Show that the flow is maximum when it is critical flow for a given specific energy plot the graph " E_0" verses" q"

Solution:

$$\text{Specific energy} = E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$

when " α = 1.0 "

$$\therefore\ Q = \sqrt{2gA^2(E\text{-}y)} = A\ .\ \sqrt{2g(E - y)}$$

For the flow to be maximum, " $\frac{dQ}{dy} = 0$ "

$$\frac{dQ}{dy} = \frac{d}{dy}\left[A\sqrt{2g(E - y)}\right] = 0$$

$$= \sqrt{2g(E - y)}\frac{dA}{dy} + A\sqrt{2g}.\frac{d}{dy}(E\text{-}y)^{\frac{1}{2}} = 0$$

$$= \sqrt{2g(E - y)}\frac{dA}{dy} - \frac{A\sqrt{2g}}{2\sqrt{E\text{-}y}} = 0$$

$$\sqrt{2g(E\text{-}y)}\frac{dA}{dy} = \frac{A\sqrt{2g}}{2\sqrt{E\text{-}y}}$$

$$2(E\text{-}y)\frac{dA}{dy} = A \qquad (1)$$

But $Q = A\sqrt{2g(E - y)}$

$$=> 2(E - y) = \frac{Q^2}{gA^2}$$

Substituting in eqn: (1) and taking $\frac{dA}{dy} = T$,

$$\frac{Q^2T}{gA^2} = A$$

$$=> \frac{Q^2T}{gA^3} = 1$$

But $\frac{Q^2T}{gA^3} = \frac{V^2T}{gA} = \frac{V^2}{gD}$

$$\therefore \frac{V^2}{gD} = 1$$

But $\frac{V^2}{gD} = F^2$

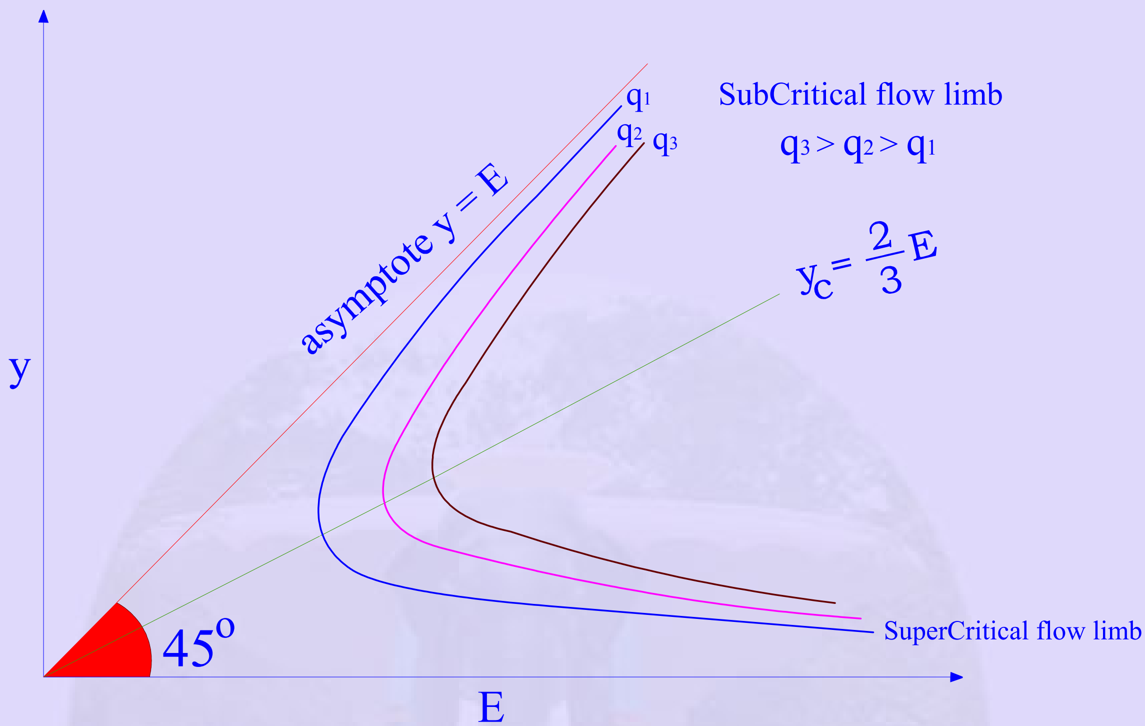
=> F^2 =1

=> F = 1 i.e., Flow is critical. Flow is maximum for a given specific energy, when it is in critical state.

Which is essentially equation representing the critical flow. Thus critical flow connotes not only minimum specific energy for a given discharge per unit width, but also maximum discharge per unit width for given specific energy.

Any one of the above three equations may be used to define critical flow. For example:

(1) The crests of E-y curves drawn for all values of q can be joined by a straight line having the equation y = 2E / 3, as shown in Figure.



(2) y_c increases with q. The curves of higher value of q are to the right of a curve with a low value of q.

For a given q and if the slope θ is small than $y \to 0,\ E \to \infty,$ an asymptote.

Similarly y = E is another asymptote. The specific energy equation can be written as

$$\left(E - y\right) y^2 = \frac{q^2}{2g} = \text{a constant}$$

For a given specific energy and q there are three routes for depth - two of them are real and one imaginary. These supercritical and sub critical depths are called alternate depths.