11.1 Critical depth in Trapezoidal and Circular channels

Problem:

For trapezoidal channel show that $\dfrac{Q^2 m^3}{g b^5} = \dfrac{y_c^{'3}\left(y_c^{'}+1\right)^3}{2y_c^{'}+1}$, *where* $y_c^{'} = \dfrac{my_c}{b}$

Solution:

The most important basic problem is to determine the critical depth.

From the dimensional analysis

$$y_c = f\left(Q, b, g, m\right)$$

Combining *it can be rewritten as* $\dfrac{y_c}{b} = f\left(\dfrac{Q^2}{gb^5}, m\right)$

For critical flow $F = \dfrac{\overline{V}}{\sqrt{gD}} = 1.$

$$\therefore \frac{\overline{V}^2}{g} = D$$

From continuity equation $\overline{V}^2 = \dfrac{Q^2}{A^2}$

$$A^2 D = \frac{Q^2}{g}.$$

$$\therefore \text{Section factor } Z = \frac{Q}{\sqrt{g}}$$

Consider $A = \left(b + my_c\right) y_c$

$$= \left(1 + \frac{my_c}{b}\right) by_c$$

$$T = \left(1 + \frac{2my_c}{b}\right) b$$

$$\frac{A}{T} = \frac{\left(1 + \frac{my_c}{b}\right) y_c}{\left(1 + \frac{2my_c}{b}\right)}$$

$$Z^2 = A^2 D = \left[\left(1 + \frac{my_c}{b}\right) by_c\right]^2 \left[\frac{\left(1 + \frac{my_c}{b}\right) y_c}{\left(1 + \frac{2my_c}{b}\right)}\right]$$

Defining $y_c^{'} = \dfrac{my_c}{b}$

$$Z^2 = \frac{\left(1 + y_c^{'}\right)^2 \left(1 + y_c^{'}\right)\left(y_c^3 b^2\right)}{\left(1 + 2y_c^{'}\right)} = \frac{Q^2}{g}$$

Multiplying *on both sides by* $\dfrac{m^3}{b^5}$ *we get*

$$\frac{Q^2}{g}\frac{m^3}{b^5} = \frac{\left(1 + y_c^{'}\right)^2 \left(1 + y_c^{'}\right)\left(y_c^3 b^2\right)}{\left(1 + 2y_c^{'}\right)}\left(\frac{m^3}{b^5}\right) = \frac{\left(1 + y_c^{'}\right)^2 \left(1 + y_c^{'}\right)}{\left(1 + 2y_c^{'}\right)} y_c^{'3}$$

$$\frac{Q^2}{g}\frac{m^3}{b^5} = \frac{\left(1 + y_c^{'}\right)^3 y_c^{'3}}{\left(1 + 2y_c^{'}\right)}$$

Problem:

Show that for circular channel

$$\left\{\frac{\mathbf{Q}}{\mathbf{d_o^2}\sqrt{\mathbf{gd}_o}}\right\}^2 = \frac{\beta - \sin\beta\ \cos\beta}{64\ \sin\beta}$$

In which “y” is the depth of flow " d_0" is the diameter of the circular channel.

Solution:

When flow is critical,

$$\text{Froude number } \mathbf{F} = \frac{\mathbf{V}}{\sqrt{\mathbf{gD}}} = 1$$

$$\Rightarrow \mathbf{V} = \sqrt{\mathbf{gD}} = \frac{\mathbf{Q}}{\mathbf{A}}$$

$$\Rightarrow \frac{\mathbf{Q^2}}{\mathbf{A^2}} = \mathbf{gD} \quad ---------->(1)$$

$$\text{Area of flow } \mathbf{A} = \frac{\mathbf{r^2}}{2}(\theta - \sin\theta) = \frac{\mathbf{d_0^2}}{8}(\theta - \sin\theta)$$

substituting $\theta = 2\beta$ **it can be written as**

$$\mathbf{A} = \frac{\mathbf{d_0^2}}{8}(2\beta - \sin 2\beta)$$

$$\text{Top width} = \mathbf{T} = 2\left[\frac{\mathbf{d}_o}{2}\sin(\pi - \beta)\right] = \mathbf{d}_o \sin(\pi - \beta) = \mathbf{d}_o \sin\beta$$

$$\text{Hydraulic depth } \mathbf{D} = \frac{\mathbf{A}}{\mathbf{T}} = \frac{\mathbf{d}_o}{8}\left[\frac{(2\beta - \sin 2\beta)}{\sin\beta}\right] = \frac{\mathbf{d}_o}{8}\left[\frac{(2\beta - 2\sin\beta\cos\beta)}{\sin\beta}\right]$$

$$\mathbf{D} = \frac{\mathbf{d}_o}{4}\left[\frac{(\beta - \sin\beta\cos\beta)}{\sin\beta}\right]$$

$$\text{From eq: (1)} \qquad \mathbf{gD} = \frac{\mathbf{Q^2}}{\mathbf{A^2}} = \frac{\mathbf{Q^2}}{\frac{\mathbf{d}_o^4}{64}\left[(2\beta - 2\sin\beta\cos\beta)^2\right]}$$

$$\mathbf{gD} = \frac{\mathbf{gd}_o}{4}\left[\frac{\beta - \sin\beta\cos\beta}{\sin\beta}\right] = \frac{\mathbf{Q^2}}{\frac{\mathbf{d}_o^4}{16}\left[(\beta - \sin\beta\cos\beta)^2\right]}$$

$$\therefore \frac{\mathbf{Q^2}}{\frac{\mathbf{d}_o^4}{16}\left[(\beta - \sin\beta\cos\beta)^2\right]} = \frac{\mathbf{g\ d}_o}{4}\left[\frac{\beta - \sin\beta\cos\beta}{\sin\beta}\right]$$

$$\Rightarrow \frac{\mathbf{Q^2}}{\mathbf{gd}_o^5} = \frac{1}{64}\left[\frac{(\beta - \sin\beta.\cos\beta)^3}{\sin\beta}\right]$$

$$\therefore \frac{\mathbf{Q^2}}{\mathbf{d}_o^2\sqrt{\mathbf{gd}_o}} = \frac{1}{64\ \sin\beta}(\beta - \sin\beta\cos\beta)^3$$