11.2 Hydraulic exponent for critical flow: M

The section factor Z for critical flow in general can be expressed as

$$Z^2 = C_o y_c^M$$

in which M is an exponent to be determined and C_0 is a constant proportionality.

Taking logarithm on both sides

$$2 \ln Z = \ln C_o + M \ln y_c$$

Differentiating with respect to y

$$2\frac{d}{dy}(\ln Z) = \frac{d}{dy}(\ln C_o) + M\frac{d}{dy}(\ln y_c)$$

$$\frac{d}{dy}(\ln Z) = \frac{M}{2 y_c} \qquad (1)$$

But from definition $Z = \frac{A\sqrt{A}}{\sqrt{T}}$

Taking logarithm it may be expressed as

$$\ln Z = \ln\left(A^{3/2}\right) + \ln\left(T^{-1/2}\right)$$

$$\frac{d}{dy}(\ln Z) = \frac{3}{2}\frac{d}{dy}(\ln A) - \frac{1}{2}\frac{d}{dy}(\ln T)$$

$$\frac{d}{dy}(\ln Z) = \frac{3}{2}\frac{1}{A}\frac{dA}{dy} - \frac{1}{2T}\frac{dT}{dy} \qquad (2)$$

Comparing equations (1) and (2) it may be written as

$$M = 2 y_c\left[\frac{3}{2}\frac{1}{A}\frac{dA}{dy} - \frac{1}{2T}\frac{dT}{dy}\right]$$

But $\frac{dA}{dy} = T$, then

$$M = y_c\left[3\frac{T}{A} - \frac{1}{T}\frac{dT}{dy}\right]$$

$$M = \frac{y_c}{A}\left[3T - \frac{A}{T}\frac{dT}{dy}\right]$$

M is known as Hydraulically exponent for critical flow.

It may be noted that no particular channel shape has been assumed.

(a) If channels of rectangular cross section,

$$\frac{dT}{dy} = 0$$

$$\therefore M = \frac{3 y_c T}{b y_c}$$

$$M = 3.0 \;(\because\; T = b)$$

(b) For trapezoidal channel obtain the following expression

$$M = 3\frac{\left(1+2y_c'\right)}{\left(1+y_c'\right)} - \frac{2y_c'}{\left(1+2y_c'\right)}$$

in which $y_c' = m\frac{y_c}{b}$.

Solution :

For trapezoidal channel (*for critical flow*)

$$A = (b+my)y_c, \quad T = b+2my_c, \qquad \frac{dT}{dy} = 2m.$$

Substituting the above in the standard expression for $M = \frac{y_c}{A}\left[3T - \frac{A}{T}\frac{dT}{dy}\right]$

$$M = \frac{y_c}{(b+my_c)y_c}\left[3(b+2my_c) - \frac{(b+my_c)y_c}{(b+2my_c)}2m\right]$$

$$M = \frac{1}{b\left(1+\frac{my_c}{b}\right)}\left[3b\left(1+\frac{2my_c}{b}\right) - \frac{b\left(1+\frac{my_c}{b}\right)2my_c}{b\left(1+\frac{2my_c}{b}\right)}\right]$$

$$M = \frac{1}{b\left(1+\frac{my_c}{b}\right)}\left[3b\left(1+\frac{2my_c}{b}\right) - \frac{b\left(1+\frac{my_c}{b}\right)\frac{2my_c}{b}}{\left(1+\frac{2my_c}{b}\right)}\right]$$

$$M = \frac{1}{\left(1+\frac{my_c}{b}\right)}\left[\frac{3\left(1+\frac{2my_c}{b}\right)\left(1+\frac{2my_c}{b}\right) - \left(1+\frac{my_c}{b}\right)\frac{2my_c}{b}}{\left(1+\frac{2my_c}{b}\right)}\right]$$

If $y_c' = \frac{my_c}{b}$

$$M = \frac{1}{\left(1+y_c'\right)}\left[\frac{3\left(1+2y_c'\right)\left(1+2y_c'\right) - \left(1+y_c'\right)2y_c'}{\left(1+2y_c'\right)}\right]$$

$$M = \left[\frac{3\left(1+2y_c'\right)^2 - 2\left(1+y_c'\right)y_c'}{\left(1+y_c'\right)\left(1+2y_c'\right)}\right]$$

$$M = 3\frac{\left(1+2y_c'\right)}{\left(1+y_c'\right)} - \frac{2y_c'}{\left(1+2y_c'\right)}$$

Show for Triangular channel $M = 5.0$

Solution :

Section factor $Z = A\sqrt{D} = my^2\left(\sqrt{\frac{y}{2}}\right)$

$$Z^2 = \left[my^2\left(\sqrt{\frac{y}{2}}\right)\right]^2 = m^2y^4\left(\frac{y}{2}\right) = \frac{m^2}{2}y^5$$

$$Z^2 = Cy^M$$

Comparing these two equations $M = 5$

Critical flow exponent for non prismatic channel (Nature channel) :

$$\tan\theta = \frac{\log Z_2 - \log Z_1}{\log y_2 - \log y_1}$$

$$\tan\theta = \frac{\log\left(\frac{Z_2}{Z_1}\right)}{\log\left(\frac{y_2}{y_1}\right)}$$

$$Z^2 = Cy_c^M$$

$$2\ln Z = \ln C + M\ln y_c$$

$$\therefore M = 2\frac{\ln Z}{\ln y_c} = 2\tan\theta$$

$$M = 2\tan\theta$$

$$2\ln Z_1 = \ln C + M\ \ln y_{c_1}$$

$$2\ln Z_2 = \ln C + M\ \ln y_{c_2}$$

Subtracting

$$2\ln Z_1 - 2\ln Z_2 = M\left[\ln y_{c_1} - \ln y_{c_2}\right]$$

$$2\ln\left(\frac{Z_1}{Z_2}\right) = M\ \ln\left[\frac{y_{c_1}}{y_{c_2}}\right]$$

$$M = \frac{2\ln\left(\frac{Z_1}{Z_2}\right)}{\ln\left[\frac{y_{c_1}}{y_{c_2}}\right]} = 2\tan\theta$$

(C) It may be noted that by using $\frac{my}{b}$ Vs M, a single curve can be constructed. Then this curve could be identical to the curve with $m = 1.0$

Similarly the graph for $\frac{y_c}{d_0}$ Vs $\frac{Q}{D^2\sqrt{gD}}$ can be constructed.