

11.3 Problem

Derive the value of M, N for rectangular (narrow, wide), Trapezoidal, Triangular channel by using the following expression.

$$M = \frac{y}{A} \left(3T - \frac{A}{T} \frac{dT}{dy} \right) \text{ and } N = \frac{2y}{3A} \left(5T - 2R \frac{dP}{dy} \right)$$

Solution:

(a) Rectangular Channel

$$\begin{aligned} M &= \frac{y}{A} \left(3T - \frac{A}{T} \frac{dT}{dy} \right) \\ &= \frac{y}{by} \left(3b - \frac{by}{b} (0) \right) \\ &= \frac{y}{by} (3b) \end{aligned}$$

$$M = 3.0$$

For wide rectangular channel

$$A=by, R \rightarrow y, p \rightarrow b \frac{dp}{dy} = 0$$

$$\begin{aligned} N &= \frac{2y}{3A} \left(5T - 2R \frac{dp}{dy} \right) \\ &= \frac{2y}{3by} \left(5b - 2 \frac{by}{b+2y} (0) \right) \\ &= \frac{2y}{3by} (5b) \\ &= 10/3 = 3.33 \end{aligned}$$

For narrow channel

$$R = \frac{b}{2} \text{ as } b \rightarrow 0, p = b + 2y \frac{dp}{dy} = 2$$

$$\begin{aligned} N &= \frac{2}{3} \frac{y}{by} \left[5b - 2 \frac{b}{2} \cdot 2 \right] \\ &= \frac{2}{3} \frac{y}{by} [5b - 2b] = 2.0 \end{aligned}$$

For Chezy relationship

$$\begin{aligned} N &= \frac{y}{A} \left(3T - R \frac{dp}{dy} \right) \\ N &= \frac{y}{by} \left(3b - \frac{by}{b+2y} (0) \right) \\ N &= 3.0 \end{aligned}$$

(b) Triangular Channel:

1) Area of triangle

$$A = my^2$$

2) $P = 2y\sqrt{1+m^2}$

3) $R = \frac{my}{2\sqrt{1+m^2}}$

4) $T = 2my$

$$M = \frac{y}{A} \left(3T - \frac{A}{T} \frac{dT}{dy} \right)$$

$$= \frac{y}{2y*y} \left(3 \cdot 2my - \frac{my*y}{2my*y} \left(\frac{2my}{y} \right) \right)$$

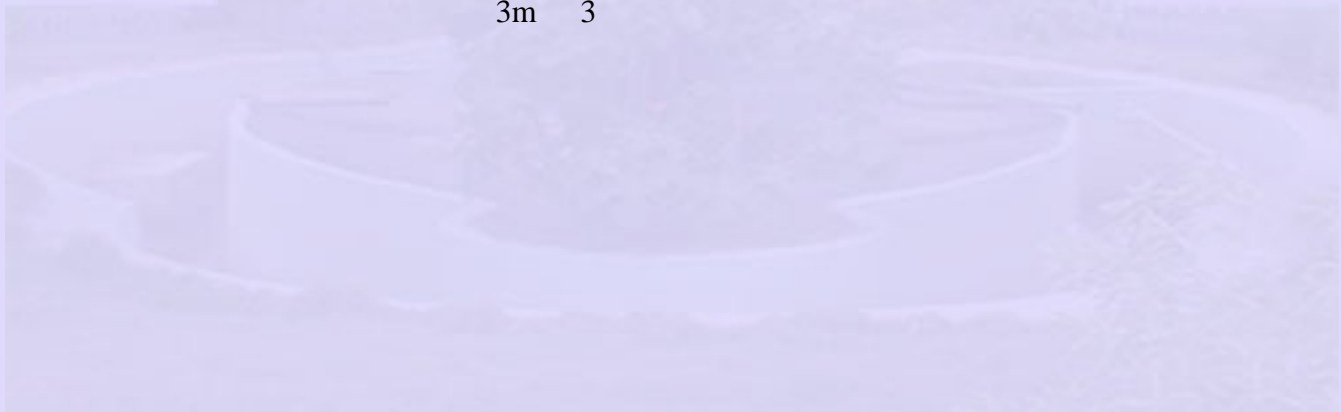
$$= \frac{y}{my*y} (5my) = 5$$

$$N = \frac{2y}{3A} \left[5 * 2my - 2 \frac{my}{2\sqrt{1+m^2}} 2\sqrt{1+m^2} \right]$$

$$= \frac{2y}{3my^2} \left[10my - \frac{my}{\sqrt{1+m^2}} 2\sqrt{1+m^2} \right]$$

$$= \frac{2}{3} \frac{y^2}{my^2} [10m - 2m]$$

$$= 8m * \frac{2}{3m} = \frac{16}{3}$$



(c) Trapezoidal Channel

$$\begin{aligned}
 M &= \frac{y}{A} \left(3T - \frac{A}{T} \frac{dT}{dy} \right) \\
 &= \frac{y}{(b+my)y} \left[3(b+2my) - \frac{(b+my)y}{(b+2my)} \frac{d(b+2my)}{dy} \right] \\
 &= \frac{y}{(b+my)y} \left[3(b+2my) - \frac{(b+my)y}{(b+2my)} 2m \right] \\
 &= \frac{y}{(b+my)y} \left[\frac{3(b+2my)^2 - (b+my)2my}{(b+2my)} \right] \\
 &= \frac{3(b+2my)^2 - (b+my)2my}{(b+my)(b+2my)^2} \\
 &= \frac{3b^2 \left[1 + 2\frac{my}{b} \right]^2 - \left(1 + \frac{my}{b} \right) 2\frac{my}{b} b}{\left(1 + \frac{my}{b} \right) by \left[1 + 2\frac{my}{b} \right] b} \\
 M &= \frac{3 \left[1 + 2\frac{my}{b} \right]^2 - \left(1 + \frac{my}{b} \right) 2\frac{my}{b}}{\left(1 + \frac{my}{b} \right) \left[1 + 2\frac{my}{b} \right]}
 \end{aligned}$$

$$\text{if } y'_c = \frac{my_c}{b}$$

$$M = \frac{3 \left[1 + 2y'_c \right]^2 - \left(1 + y'_c \right) 2y'_c}{\left(1 + y'_c \right) \left[1 + 2y'_c \right]}$$

$$\begin{aligned}
 N &= \frac{2y}{3A} \left(5T - 2R \frac{dP}{dy} \right) \\
 &= \frac{2y}{3(b+my)y} \left(5 * (b+2my) - 2 \frac{(b+my)y}{b+my\sqrt{1+m^2}} \right) - 2\sqrt{1+m^2} \\
 N &= \frac{10 \left(1 + 2m \frac{y}{b} \right) - \frac{8 \frac{y}{b} \sqrt{1+m^2}}{3 \left(1 + 2 \frac{y}{b} \sqrt{1+m^2} \right)}}{3 \left(1 + m \frac{y}{b} \right)} \\
 N &= \frac{10 \left(1 + 2y' \right) - \frac{8 \frac{y}{b} \sqrt{1+m^2}}{3 \left(1 + 2 \frac{y}{b} \sqrt{1+m^2} \right)}}{3 \left(1 + y' \right)} \quad \text{in which } y' = \frac{my_n}{b}
 \end{aligned}$$