12.1 Critical flow depth computations

One of the important aspects in Hydraulic Engineering is to compute the critical depth if discharge is given.

Following methods are used for determining the critical depth.

(i) Algebraic method.

(ii) Graphical method.

(iii) Design chart.

(iv) Numerical method. Bi section method/ Newton Raphson method.

(v) Semi empirical approach - a method has introduced by Strarb.

12.1.1 Algebraic method

In this method the algebraic equation is formulated and then solved by trial and error. The following example illustrates the method.

1. Consider a trapezoidal channel:
2.

$$A = (b + my_c) y_c$$

$$D = \frac{(b + my_c) y_c}{(b + 2my_c)}$$

$$Z_c = \frac{Q}{\sqrt{g}} = \text{ constant} = C_1 = \text{known}$$

$$C_1 = (b + my_c) y_c \left\{ \frac{(b + my_c) y_c}{(b + 2my_c)} \right\}^{1/2} \qquad (1)$$

$$C_1^2 (b + 2my_c) = (b + my_c)^3 y_c^3$$

leads to

$$y_c^6 + py_c^5 + qy_c^4 + ry_c^3 + sy_c + t = 0$$

in which the constants p, q, r, s and t are known.

Solve this by polynomial or by trial and error method.

It would be easier to solve the equation (1) by trial and error procedure.

After obtaining the answer check for the Froude number which should be equal to 1.

Example:

Consider a Rectangular channel and obtain the critical depth for a given discharge.

Solution:

$$\text{Area} = b\,y \qquad D = \frac{A}{T} = \frac{b\,y}{b} = y$$

$$\therefore\ Z = \frac{Q}{\sqrt{g}} = b\,y\,y^{1/2}$$

$$y_c^{3/2} = \frac{Q}{b\sqrt{g}}$$

$$y_c = \left(\frac{Q}{b\sqrt{g}}\right)^{2/3} = \sqrt{\left(\frac{q}{\sqrt{g}}\right)^2} = \sqrt[3]{\frac{q^2}{g}}$$

12.1.2 Trial and error method

For a given trapezoidal channel obtain the critical depth by trial and error method.

Solution:

For trapezoidal channel

$$A\sqrt{D} = \frac{\left[(b+my_c)\,y_c\right]^{3/2}}{(b+2my_c)^{1/2}}$$

$$\text{Squaring}\ \left(\frac{(b+my)\,y}{(b+2my)}\right)^3 y^3 = \frac{Q^2}{g} = \text{constant}$$

For a given b, m, Q, select a value of y_c

Assume b = 6 m, m = 2m, Q = 12 m^3/s Solve for y_c

$$\frac{(6+2y_c)^3\,y_c^3}{6+4y_c} = \frac{144}{9.81} = 14.679$$

$$\frac{(3+y_c)^3\,y_c^3}{3+2y_c} = \frac{36}{9.81} = 3.6697$$

Assume a value of y_c and compute $A\sqrt{D}$ and compare with the value obtained by $\frac{Q}{\sqrt{g}}$.

y_c	A	D	$A\sqrt{D}$	Remarks
1.2			23.708	too high
0.5			1.339	low
0.8			6.170	high
0.65			3.10	
0.70			3.94	

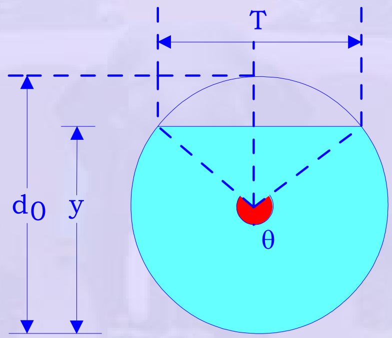
Remarks column indicate that the values are high or low when compared to the given value. The improvement is done till it converges.

In the above table y_c lies between 0.65 and 0.70.

This could be improved further by selecting the values in between these two.

12.1.3 Graphical method

For natural channels and complicated channels, the graphical method is adopted. A curve is generated assuming different values of y_c and Z. The value of $\frac{Q}{\sqrt{g}}$ is computed and y_c is obtained from the chart. A one meter diameter culvert carries a discharge of 0.7 m^3/s. Determine the critical depth.



$$D = \frac{1}{g}\left[\frac{\theta - \sin\theta}{\sin\frac{\theta}{2}}\right] d_o$$

$$Z = \frac{\sqrt{2}\left(\theta - \sin\theta\right)^{1.5}}{32\left[\sin\frac{\theta}{2}\right]^{0.5}} d_0^{0.5}$$

Knowing the value of d_0 for different values of depth A and D could be obtained from the table.

Example:

A one meter diameter pipe carries a discharge of 0.7 m^3/s. Determine the critical depth.

$$Z_c = \frac{Q}{\sqrt{g}} = \frac{0.7}{3.132} = 0.2235$$

Construct a graph of y_c Vs Z and obtain the value of y_c

From the graph $y_c = 0.4756$

From the design chart determine the critical depth for a circular channel of 0.9 m diameter. Discharge 0.71 m^3/s.

Solution:

$$Z = \frac{0.71}{\sqrt{9.81}} = 0.22669$$

$$\frac{Z}{d_0^{2.5}} = 0.29499 \ \left(\textit{from table} \right)$$

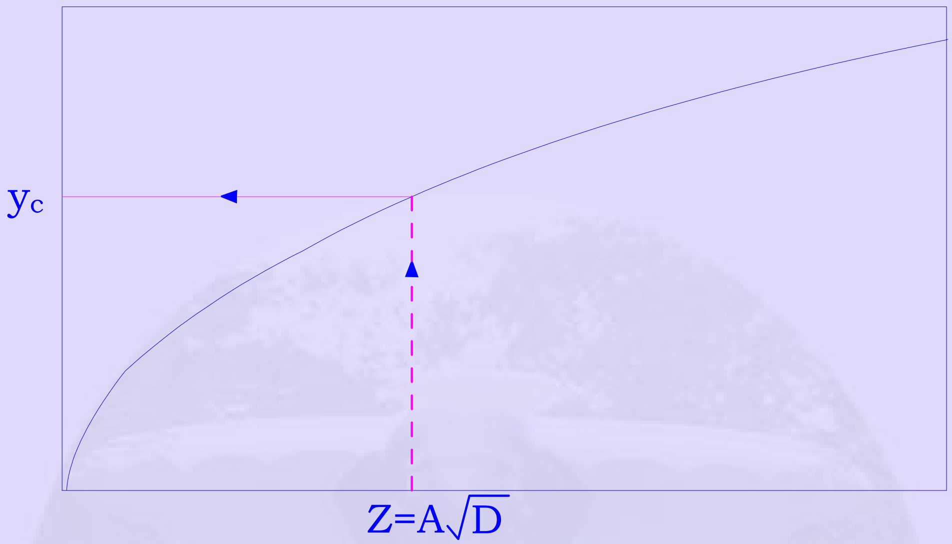
$$\frac{y_c}{d_0} = 0.56, \quad y_c = 0.49527\, m$$

$$\psi = \frac{\alpha Q^2}{g} = \frac{1.0\left(17\right)^2}{9.81} = 29.5$$

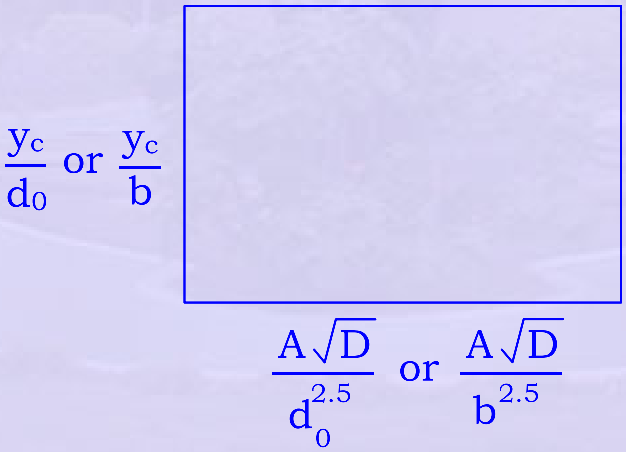
$$y_c = 0.81\left[\frac{29.5}{2^{0.75} 6^{1.25}}\right]^{0.27} - \frac{6}{30\left(2\right)} = 0.86\ m$$

12.1.4 Graphical Procedure

Straub proposed several semi empirical equations to obtain the critical depth. The advantage of this is a quick estimation of the critical depth. However, the equations are non homogenous.



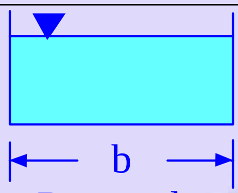
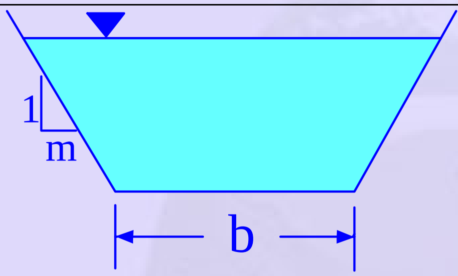
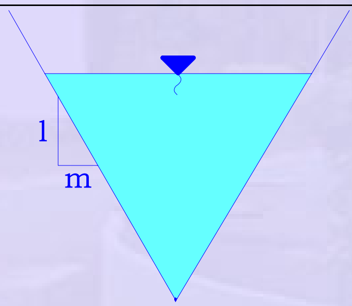
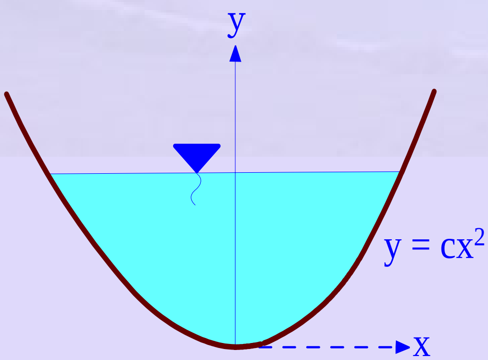
Graph showing variation of section factor with critical depth for a given pipe of diameter d_o

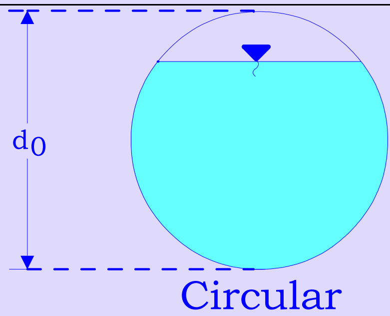
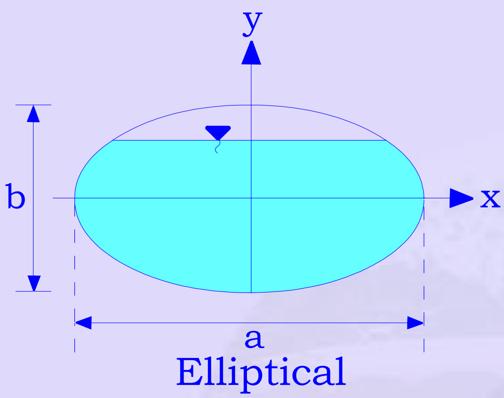
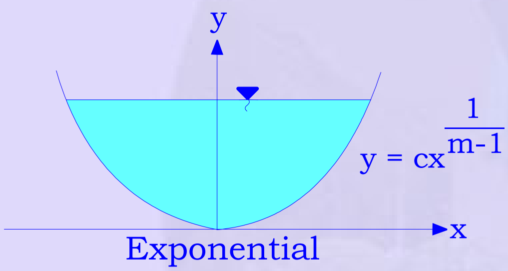


Reference:

Straub W.O, Civil Engineering, ASCE, 1978 Dec, pp 70 - 71 and Straub 1982.

Table: Semi empirical equations for the estimation of y_c (Straub, 1982) MKS units

Channel type	Equation for y_c in terms of $\psi = \alpha Q^2 / g$	
 Rectanglar	$\left(\frac{\psi}{b^2}\right)^{1/3}$	
 Trapezoidal	$0.81\left(\frac{\Psi}{m^{0.75}b^{1.25}}\right)^{0.27} - \frac{b}{30m}$	Range of applicability $0.1 < \frac{Q}{b^{2.5}} < 4.0$ For $\frac{Q}{b^{2.5}} < 0.1$ use equation for rectangular channel
 TRIANGULAR	$\left(\frac{2\Psi}{m^2}\right)^{0.20}$	
 Parabolic	$(0.84c\Psi)^{0.25}$	$y = cx^2$

	$\left(\frac{1.01}{d_0^{0.26}}\right)\Psi^{0.25}$ $y_c = 0.053\frac{Q^{0.52}}{d_0^{0.3}}$ $y_c = [m]$ $Q = m^3s^{-1}$, $d_0 = [m]$	Range of applicability $0.02 \le \frac{y_c}{d_o} \le 0.85$
	$0.84b^{0.22}\left(\frac{\psi}{a^2}\right)^{0.25}$	Range of applicability $0.05 \le \frac{y_c}{2b} \le 0.85$ a = major axis b = minor axis
	$\left(\frac{m^3\psi c^{2m-2}}{4}\right)^{1/(2m+1)}$	$y = cx^{1/(m-1)}$

Example:

$b = 6.0$ m, $m = 2$, $Q = 17 m^3/s$ determine y_c

Solution:

From table

$$y_c = 0.81\left(\frac{\psi}{m^{0.75}b^{1.25}}\right)^{0.27} - \frac{b}{30m} \qquad \text{for } 0.1 < \frac{Q}{b^{2.5}} < 4.0$$

where $\psi = \frac{\alpha Q^2}{g}$

The value of $\frac{Q}{b^{2.5}} = \frac{17}{6^{2.5}} = 0.19$,

It is in the range of the equation. Substituting the appropriate values,

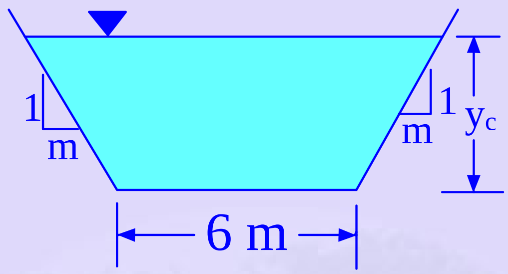
$$\psi = \frac{1(17)^2}{9.8} = 29.5$$

$$y_c = 0.81\left(\frac{29.5}{2^{0.75}6^{1.25}}\right)^{0.27} - \frac{6}{30(2)} = 0.86\ m$$

Problem:

Non rectangular channel involves trial and error solution.

Obtain the critical depth for the trapezoidal channel of bottom width 6 m with a side slope of 2.5: 1, which carries a discharge of 20 m^3/s.



Solution:

Trial and error procedure

$$A=(b+my)\,y=(6+2.5\,y_c)\,y_c$$
$$T=b+2my=6+5y_c$$
$$D=\frac{A}{T}=\frac{(6+2.5y_c)\,y_c}{6+5y_c}$$
$$Z_c=\frac{Q}{g}=A\sqrt{D}$$
$$\frac{V^2}{2g}=\frac{Q^2/A^2}{2g}=\frac{20*20}{[6+2.5\,y_c]^2\;19.62}=\quad(6+2.5y_c)\,y_c\left\{\frac{(6+2.5y_c)\,y_c}{6+5y_c}\right\}^{0.5}$$
$$y_c=\;?$$
$$v_c=\sqrt{gy_c}=?$$

Solution of Algebraic or Transcendental Equations by the Bisection Method

In the algebraic expression F(x) =0, when a range of values of x is known that contains only one root, the bisection method is a practical way to obtain it. It is best shown by an example.

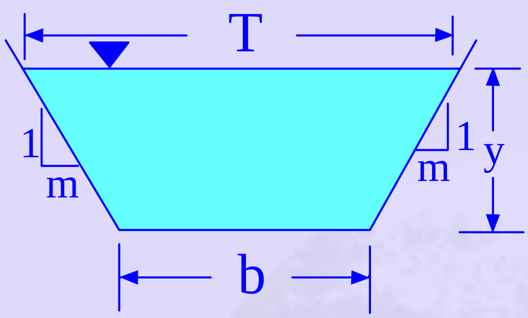
The critical depth in a trapezoidal channel is to be determined for given flow Q and channel dimensions.

$$1-\frac{Q^2T}{gA^3}=0$$

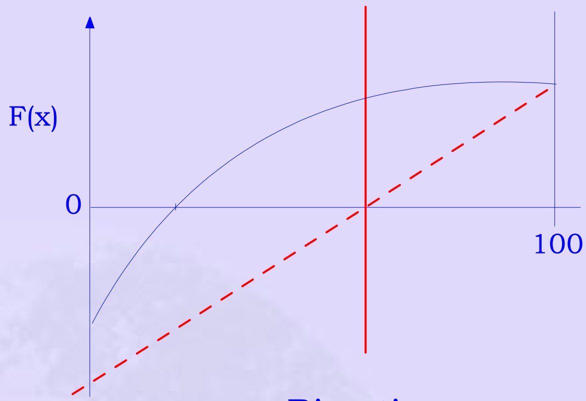
The formula must be satisfied by some positive depth y_c greater than 0 (a lower bound) and less than, an arbitrarily selected upper bound say, 10 m.

T is the free surface width $b + 2my_c$. The interval is bisected and this value of y_c tried. If the value is positive, then the root is less than the midpoint and the upper limit is moved to the midpoint and the remaining half bisected, etc.

This method gives the solution very quickly.



Trapezoidal



Bisection

Newton Raphson Method is discussed elsewhere.