12.2 Problems

There are three types of problems in critical flows as shown in table.

Type	m	Q	y_c	b or d
I	✓	?	✓	✓
II	✓	✓	?	✓
III	✓	✓	✓	?

Types I and II are easy to solve.

Type III problem requires a different approach.

Type I problem

$\frac{my_c}{b}$ is known.

$\therefore$ From the graph y_c Vs $Z = \frac{Qm^{3/2}}{b^2\sqrt{gb}}$

Q can be determined.

Type II problem

Here the solution is for obtaining critical depth. There are different methods. Graphically

$Z = \frac{Qm^{3/2}}{b^2\sqrt{gb}}$ can be computed and value of $\frac{my_c}{b}$ can be obtained from which y_c can be computed.

Type III problem

This problem can be solved using simultaneous solution of two algebraic equations which is illustrated below.

Defining

$$Y_1 = \frac{my_c}{b} \text{ (for trapezoidal channel) or } \frac{y_c}{d_0} \text{ (for circular channel).}$$

$$\text{and } X_1 = \frac{Qm^{3/2}}{b^2\sqrt{gb}} \text{ or } \frac{Q}{d_0^2\sqrt{gd_o}}$$

Then

$$b = \frac{my_c}{Y_1}$$

$$X_1 = \frac{Q\, m^{3/2}}{\frac{m^2\, y_c^2}{Y_1^2}\sqrt{\frac{g\left(m y_c\right)}{Y_1^{1/2}}}} = \frac{Q\, m^{3/2} Y_1^{5/2}}{m^2\, y_c^2\, m^{1/2}\, y_c^{1/2}\sqrt{g}}$$

$$= \frac{Q\, Y_1^{5/2}}{m\, y_c^2\, \sqrt{g y_c}} = M_1 Y_1^{5/2}$$

$$\boxed{X_1 = M_1 Y_1^{5/2}}$$

In which

$$M_1 = \frac{Q}{m y_c^2 \sqrt{g y_c}}$$

Given Q, y_c, m,

$$X_1 = \frac{Q m^{3/2}}{b^2\sqrt{gb}}, \qquad Y_1 = \frac{m y_c}{b}$$

$$\frac{Q^2 m^3}{g b^5} = \frac{y_c'^3\left(1 + y_c'\right)^3}{1 + 2 y_c'}$$

$M_1 = \dfrac{Q}{m y_c^2 \sqrt{g y_c}}$ and is known.

$$X_1 = M_1 Y_1^{5/2}$$

Substituting in the above equation

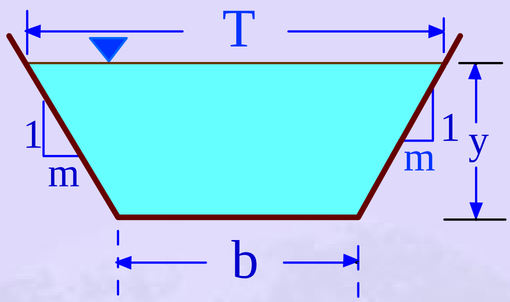
$$\frac{Q m^{3/2}}{b^2\sqrt{gb}} = M_1\left(\frac{m y_c}{b}\right)^{5/2}$$

$$\frac{Q m^{3/2}\left(m y_c^2\right)\sqrt{g y_c}}{Q\sqrt{g}} = b^{2.5} y_1^{5/2}$$

Solve the equation and obtain *the* solution for bed width b for trapezoidal channel. Similarly solve for diameter for the pipe line.

Problem

Compute the critical depth in a trapezoidal channel for flow of 30 $m^3 s^{-1}$. The channel bottom width is 10.0 m, side slope m =2. The bottom slope is negligible and $\alpha = 1$



Trapezoidal

Solution

Given

Bottom width $b = 10m$

Sideslope $m = 2$

Flow $Q = 30\ m^3 s^{-1}$

$\alpha = 1$

Critical Depth $y_c = ?$

For finding the critical depth,

Cross sectional area of the channel $A = (b + 2\,y_C) * y_C$

$= (10 + 2\,y_C) * y_C$

Section factor $Z = A\sqrt{D}$

in which $D = A / T$

for trapezoidal channel the top width $T = (b + 2m\,y_C)$

$D = (10 + 2y_C) * y_C / (10 + 2 * 2 * y_C)$

then the section factor $Z = A\sqrt{(10 + 2\,y_C) * y_C / (10 + 2 * 2 * y_C)}$

by using the equation $A\sqrt{D} = Q / \sqrt{g}, = \frac{30}{\sqrt{9.81}} = 9.578$

Substituting all the parameters A, P, T, D, and Q in the above equation and solving for y_c **one gets**

$$\frac{\left[(10 + 2y_c)y_c\right]^{3/2}}{(10 + 4y_c)^{1/2}} = 9.578$$

$$\left[(10 + 2y_c)y_c\right]^{3/2} - 9.578(10 + 4y_c)^{1/2} = 0$$

$$\left[(10 + 2y_c)y_c\right]^{3} - 91.743(10 + 4y_c) = 0$$

by trial and error, $y_c = 0.91$ **m**

Problems

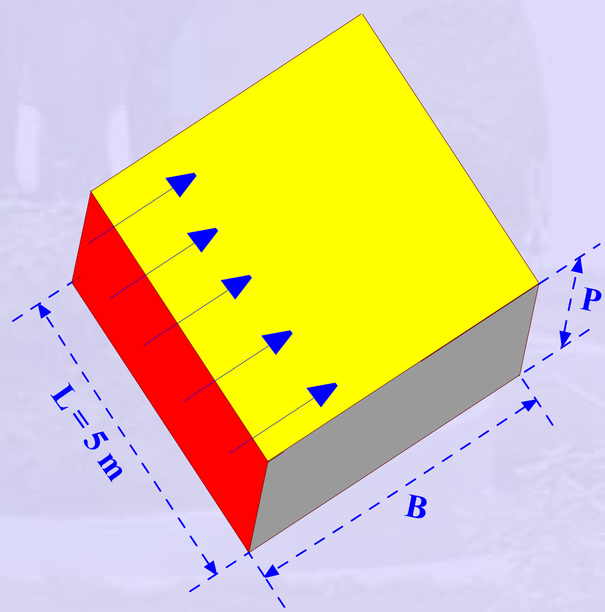
1. A trapezoidal channel with side slopes of 2 horizontal to 1 vertical is to carry a flow of 16.7 m^3/s. For a bottom width of 3.65 m, calculate (a) the critical depth and, (b) the critical velocity.

2. A rectangular channel carries 5.60 m^3/s. Find the critical depth y_c and critical velocity V_c for

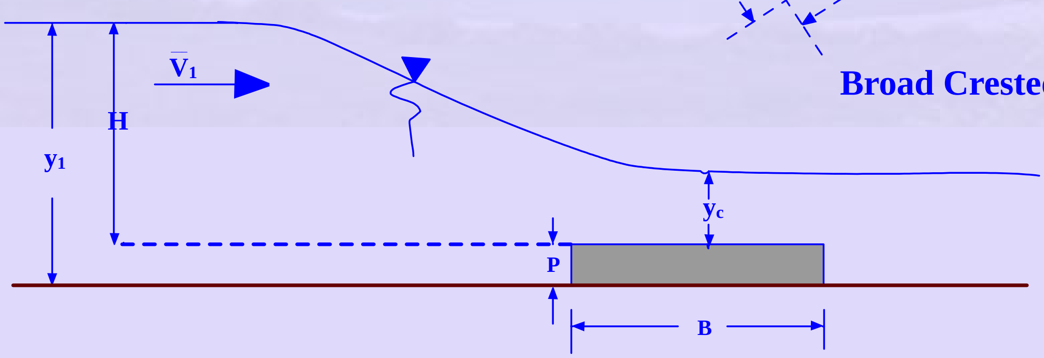
(a) a width of 3.65m and, (b) a width of 2.75m ,

(c) What slope will produce the critical velocity in (a) if n = 0.020 ?

3. Find the diacharge over a broad crested weir of 5.0m length and head 1.0m above the crest. Assume coefficient of discharge to be 0.9.



Broad Crested Weir



P is the height of weir, B is the breadth of the weir. Assume the approach velocity $\overline{\mathbf{V}}_1$ to be very small.

$$\left(\textbf{Answer:} \quad \mathbf{H} = \frac{\mathbf{3\,y_c}}{\mathbf{2}}\,, \quad \mathbf{y_c} = \sqrt{\left(\frac{\mathbf{Q}}{\mathbf{L}}\right)^2 \frac{1}{g}}, \quad \mathbf{Q} = \mathbf{0.544L}\sqrt{\mathbf{gH^{3/2}}} \right)$$