

Proportional Weirs- Introduction

The Weirs, in which the discharge is proportional to head, are known as proportional Weirs. By float-regulated dosing devices the flow over a proportional weir can be controlled. Hence are used in irrigation, in hydraulic, sanitary and chemical engineering Industry.

For a prescribed shape of weir, the discharge can be determined, e.g. in the case of a rectangular notch it is proportional to $h^{3/2}$, and in the case of a triangular (V-notch) the discharge is proportional to $h^{5/2}$, etc., where h is the head over weir. The inverse problem is for a known head-discharge relationship finding the shape of a weir constitutes the design of proportional weirs.

Linear Proportional Weir

The linear proportional weir, with its linear head-discharge characteristic is used as a control for dosing services, as a flow measuring device and as an outlet for grit chambers. The linear proportional Weir was devised by Stout (1897). This is of theoretical interest only as its width at base is infinite. This was modified by Sutro (1908) to develop a practical linear Proportional Weir and is known as the Sutro Weir (Fig Sutro Weir). A designed shape is fitted for the Sutro Weir which has a rectangular base. It is to be noted that for flows above the base of the weir, the discharge is proportional to the head measured above a reference plane located at one third of the depths of the crest of the base weir.

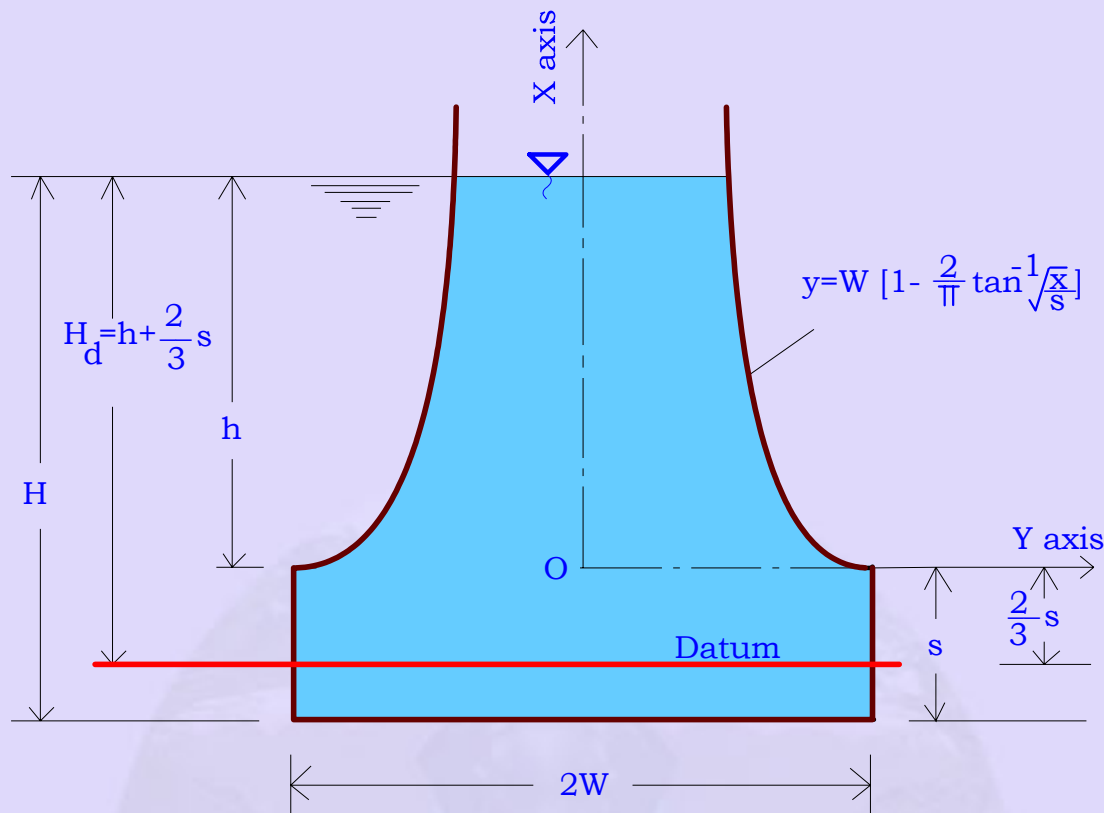


Figure- Linear proportional weir

$$\text{i.e. } Q = c \left(h + \frac{2}{3}s \right)$$

in which c is the constant of proportionality.

Shape of the Sutro Weirs

The Weir has the base of a rectangular weir of width $2W$ and height s . x and y axis are chosen as shown in figure. The Weir is assumed to be symmetrical about the ordinates axis. The discharge over the rectangular weir for a depth of flow h above the origin is

$$q_w = \frac{4}{3} WC_d \sqrt{2g} \left[(h+s)^{3/2} - h^{3/2} \right]$$

in which C_d is the coefficient of discharge.

The discharge through the upper portion above the origin (known as the complementary

weir) is $q_u = 2C_d \sqrt{2g} \int_0^h \sqrt{h-x} f(x) dx$

The total discharge through the weir is $Q = q_w + q_u$

It is required that this discharge is to be proportional to the head measured above the reference plane situated at $s/3$ above the crest of the weir. This reference plane was arbitrarily chosen by Sutro for mathematical analysis. Thus

$$Q = q_w + q_u = C_0(h + 0.67s) \text{ for } h \geq 0$$

Where C_0 is the proportionality constant.

When $h=0$, there is no flow above the base weir, hence by substituting $h=0$ in the above

equation, $C_0 = Wks^{\frac{1}{2}}$

in which $K = 2C_d\sqrt{2g}$

Substituting this value of C_0 in Equation (3) and rearranging one gets,

$$\begin{aligned} \int_0^h \sqrt{h-x} f(x) dx &= W s^{1/2} \left(h + \frac{2s}{3} \right) - \frac{2}{3} W \left[(h+s)^{3/2} - h^{3/2} \right] \\ &= W \left[\frac{2}{3} s^{3/2} + s^{1/2} h + \frac{2}{3} h^{3/2} - \frac{2}{3} (s+h)^{3/2} \right] \\ &= \frac{2}{3} W \left[h^{3/2} - \frac{3}{8} s^{-1/2} h^2 + \frac{1}{16} s^{-3/2} h^3 - \frac{3}{128} s^{-5/2} h^4 + \dots \right] \end{aligned}$$

In order that Equation (5) is satisfied for all positive values of h it is required to determine the function of $f(x)$ and can be expressed in the form of series of powers of x to determine the coefficients. A general term $f(x) = x^m$ results in

$$\begin{aligned} \int_0^h \sqrt{h-x} x^m dx &= \int_0^h x^m \left(h^{1/2} - \frac{1}{2} h^{-1/2} x + \frac{1}{8} h^{-3/2} x^2 - \dots \right) dx \\ &= C_2 (h)^{m+(3/2)} \end{aligned}$$

In which C_2 is a constant. Hence $f(x)$ can be assumed as

$$f(x) = Y = A_1 + A_2 x^{1/2} + A_3 x^{3/2} + A_4 x^{5/2} + \dots$$

Substituting this equation in Eq (5) and simplifying, it reduces to

$$\begin{aligned} f(x) &= W \left[1 - \frac{2}{\pi} \left\{ \frac{x^{1/2}}{s^{1/2}} - \frac{x^{3/2}}{3s^{3/2}} + \frac{x^{5/2}}{5s^{5/2}} - \dots \right\} \right] \\ &= W \left[1 - \frac{2}{\pi} \tan^{-1} \sqrt{\frac{x}{s}} \right] \end{aligned}$$

The discharge for the Sutro Weir is given by

$$Q = C_0 \left(h + \frac{2}{3}s \right) = C_0 \left(H - \frac{s}{3} \right) = sH_d$$

In which $C_0 = Wks^{\frac{1}{2}}$ and ($g = 9.81$), h is the head measured above the rectangular base weir and H is the total head of the flow and H_d is the head over the reference plane.

C_d ranges between 0.0597 to 0.619. Average coefficient of discharge of the Sutro Weir is 0.62.

Quadrant Plate Weir, which has the linear head-discharge relationship. This is easy to fabricate and installed under field conditions.

Example:

A sutro weir having a base of a rectangular weir of width 60cm and height of 15cm when the depth of flow is 30cm is installed in a channel. Find the discharge? If the discharge is doubled what would be the head over the weirs? Coefficient of discharge is taken as 0.62.

Solution:

a)

$$s = 150mm$$

$$W = 300mm$$

$$C_0 = WKs^{1/2} = 30 * 2 * 0.62 * \sqrt{2 * 981} * (15)^{1/2} \\ = 6381.72 \text{ cm}^2 / s$$

$$Q = C_0 \left(h + \frac{2}{3}s \right) = C_0 \left(H - \frac{s}{3} \right) \\ = C_0 H_d$$

$$Q = 6381.72 \left(30 - \frac{15}{3} \right) = 159543.2 \text{ cm}^3 / s = 159.54 \text{ l} / s$$

b)

$$Q = 319086 \text{ cm}^3 s^{-1} = 6381.72 H_d$$

$$H_d = \frac{319086}{6381.72} = 50cm$$

$$H = 50 + s/3 = 50 + 5 = 55cm$$

General Equation for the Weir

Cowgill and Banks have described the curve for the flow over weir given by

$$Q = C_0 h^m \text{ for } m \geq 0.5. \text{ The profile is } y = f(x) = \frac{c_0}{c_d \sqrt{2g\pi}} \frac{\Gamma(m+1)}{\Gamma\left(m - \frac{1}{2}\right)} (x)^{m-3/2}$$

in which Γ represents gamma function.

An attempt to design a weir producing a discharge to h^m for $m < 1.5$ inevitably leads to a curve which is asymptotic at the base giving rise to an infinite width and is not physically feasible. As discussed earlier Sutro overcame this deficiency by providing a rectangular base. A rational explanation for the selection of the datum was enunciated in the theorem of slope discharge continuity by Keshava Murthy.

The slope-discharge-continuity theorem states: "In any physically realizable weir having a finite number of finite discontinuities in its geometry, the rate of change of discharge is continuous at all points of discontinuity". In other words physically this means that the discharge head relationship for any compound weir cannot have more than one slope at any point.

The proof of this theorem is based on the use of the theorem of Laplace transformation. A known base in proportional weir is designed. Complimentary weir is provided over λ . This parameter λ known as the datum constant by the application of slope discharge-continuity theorem and is to be established for locating the reference plane or datum for any weir. The solution of integral equations helps in designing the proportional weir.

Quadratic Weir-Notch Orifice

Haszpra developed a quadratic weir in 1965. A quadratic weir is a class of proportional weir in which the discharge Q is proportional to the square root of the head h . The quadratic weir is provided with a base in the form of a rectangular weir of width $2w$ and depth s over which a designed curve is fitted.

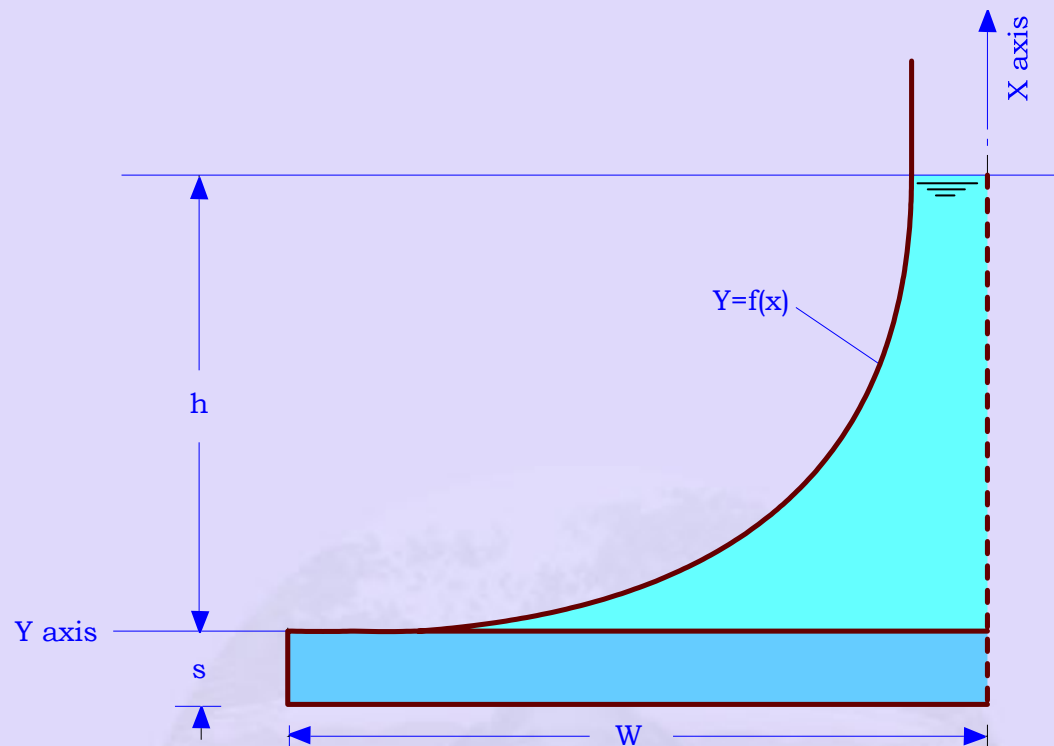


Figure: Quadratic weir with rectangular base

The weir is assumed to be sharp-edged and symmetrical over the x-axis. When the flow is h above the base, the discharge through the rectangular weir, below the y-axis, is

$$q_w = \frac{2}{3}WK \left[(h+s)^{3/2} - h^{3/2} \right]$$

in which $k = 2C_d\sqrt{2g}$, C_d is the coefficient of discharge. The discharge from the complementary weir q_c above the origin is given by

$$q_c = K \int_0^h \sqrt{h-x} f(x) dx$$

$$\text{Total discharge } Q = q_w + q_c = \frac{2}{3}WK \left[(h+s)^{3/2} - h^{3/2} \right] + K \int_0^h \sqrt{h-x} f(x) dx$$

It is to design a weir in which the discharge is proportional to the square root of the head measured above a reference plane. In other words the discharge $Q = C_1\sqrt{h+\lambda s}$, where C_1 is the proportionality constant and λ is the datum constant. These constant and are to be obtained.

They are determined using the two conditions viz;

1. Continuity of discharge and
2. The requirements of the slope discharge continuity theorem, using Leibnitz's rule for differentiating under integral sign

One may obtain $\lambda = \frac{1}{3}$ and $C_1 = \frac{2}{\sqrt{3}}WKs$

The reference plane for this weir is situated at $\frac{2}{3}s$ above the crest of the weir. Abel's form of integral equation whose solution is

$$y = f(x) = W \left[1 - \frac{2}{\pi} \tan^{-1} \sqrt{\frac{x}{s}} - \frac{6}{\pi} \frac{\sqrt{x/s}}{\left(1 + \frac{3x}{s}\right)} \right]$$

The weir beyond $=2.0 \frac{x}{s}$ acts as an orifice for all practical purposes. As the discharge is proportional to the square root of the head (measured above the reference plane), both while acting as a notch as well as an orifice, this device is also known as notch-orifice.

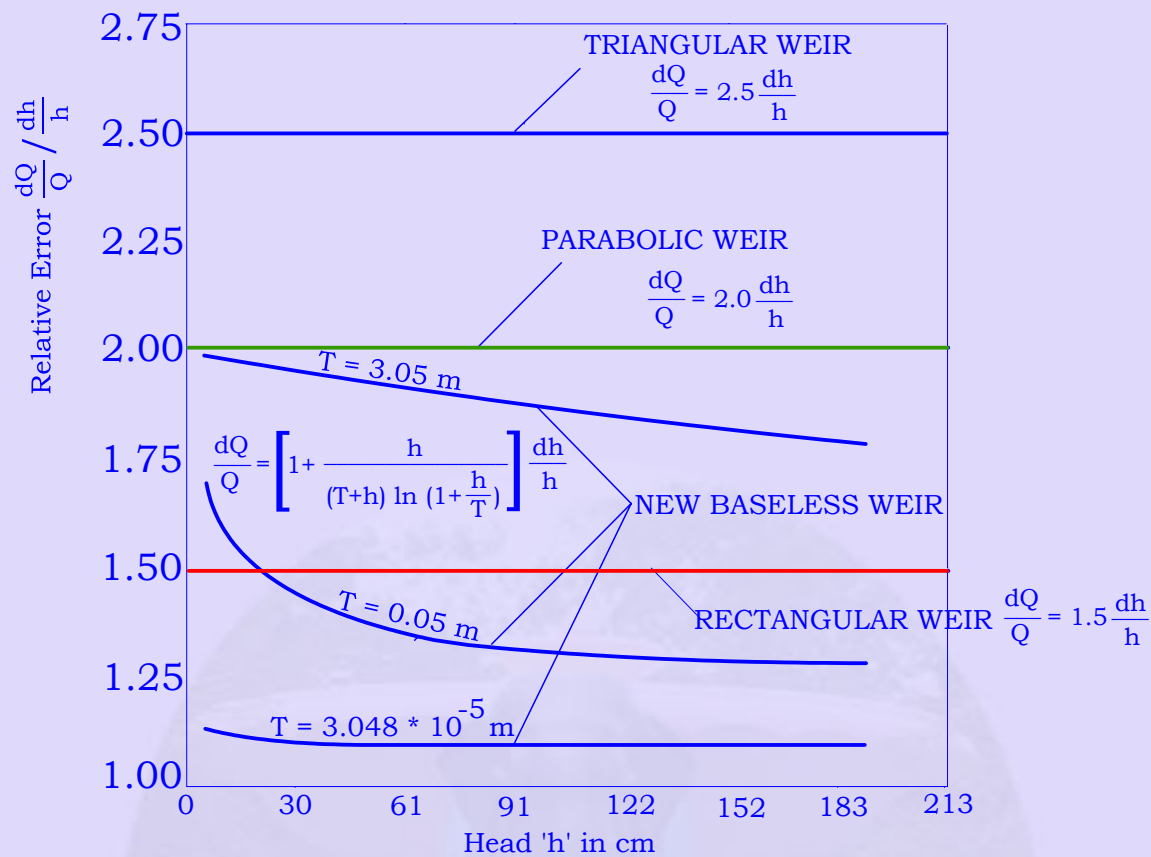
The discharge equation for the quadratic weir is

$$Q = C_1 \sqrt{h + \frac{s}{3}} = C_1 \sqrt{\left(H - \frac{2}{3}s\right)} = C_1 \sqrt{H_d}$$

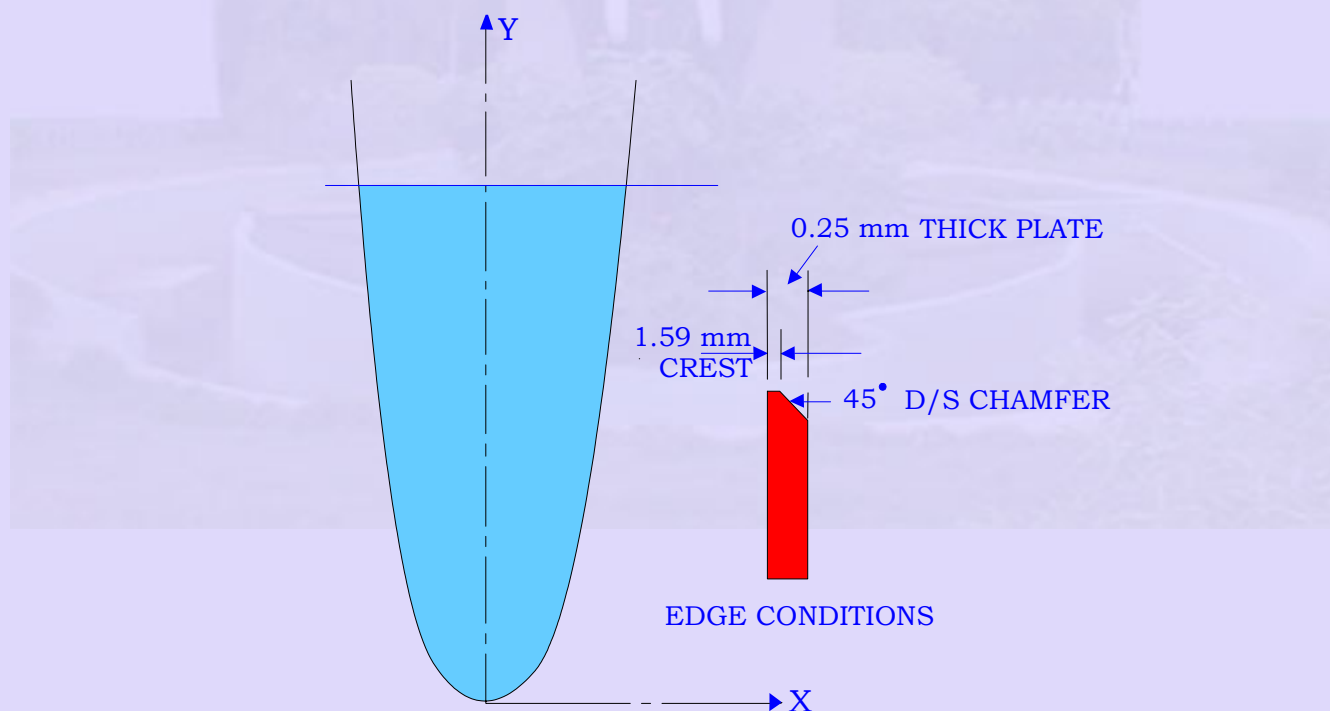
where $C_1 = \frac{2}{\sqrt{3}}WKs$ and $K = 2C_d \sqrt{2g}$

Quadratic weirs having nonrectangular lower portions (base weirs) are described by Kesavamurthy and Pillai.

An average coefficient of discharge for quadratic weir is 0.62. In a quadratic weir the error involved in the discharge computation for one percent error in head is only 0.5 percent as against 1.5% in a rectangular weir and 2.5% in a V-notch. Hence it is more sensitive than the rectangular weir and V-notch.



COMPARISON OF RELATIVE ERRORS



NEW BASELESS WEIR (NBW-1 TYPE)

Example:

If the depth of the flow is 300mm over a quadratic weir which has a 600mm width of the rectangular base weir and height of 150 mm. Determine the flow. Also find the depth for a discharge of 250 l/s. Assume C_d to be 0.62.

Solution:

A)

$$s = 150\text{mm}$$

$$W = 300\text{mm}$$

$$C_1 = \frac{2}{\sqrt{3}}WKs$$

$$= \frac{2}{\sqrt{3}} * 30 * 2 * 0.62 \sqrt{2 * 981} * 15$$

$$= 28539.92$$

$$Q = C_1 \sqrt{\left(h + \frac{s}{3}\right)} = C_1 \sqrt{H_d}$$

$$= C_1 \sqrt{\left(h - \frac{2}{3}s\right)} = 28539.92 * \sqrt{30 - \left(\frac{2}{3} * 15\right)}$$

$$Q = 127.634 \text{ l/s}$$

in which h is the depth of flow above the rectangular base, H_d is the head above the reference plane and H is the depth of flow

B)

When $Q = 250 \text{ l/s}$

$$\frac{Q_1}{Q_2} = \sqrt{\frac{H_1 - \frac{2}{3}s}{H_2 - \frac{2}{3}s}}$$

$$\sqrt{H_2 - 15} = \left(\frac{250}{127.634}\right) \sqrt{20}$$

$$H_2 = 61.732 \text{ cm}$$