18.2 Establish the conditions for maximum discharge and maximum velocity - a relation between depth and diameter using chezy equation and Manning equation as shown in table for a Circular Channel.

Show that

	Manning's equation	Chezy's equation
Maximum conveyance	$\frac{y}{d_0} = 0.938$ or $302^\circ\, 22'$	$\frac{y}{d_0} = 0.95$ or 308°
Maximum velocity	$0.81 \frac{y}{d_0}$ $256^\circ 27' 56''$	$\frac{y}{d_0} = 0.81$ $\theta = 257^\circ 27'$

Solution

Chezy equations

(a) Circular section (Maximum discharge)

$$A = \frac{r^2}{2}\,(\theta - \sin\theta)$$

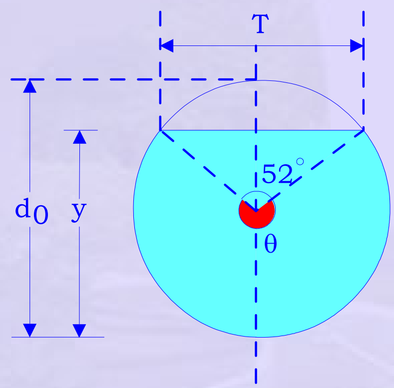
$$p = r\theta,\; Q = AC\sqrt{RS}$$

$$Q = \frac{r^2}{2}\,(\theta - \sin\theta)\; C\sqrt{RS_0}$$

$$Q = \frac{r^2}{2}\,(\theta - \sin\theta)\; C\sqrt{\frac{r}{2}\frac{(\theta - \sin\theta)}{\theta} S_0}$$

$$\left[\because R = \frac{A}{P} = \frac{\frac{r^2}{2}(\theta - \sin\theta)}{r\theta} = \frac{r}{2\theta}(\theta - \sin\theta)\right]$$

$$Q = \frac{(\theta - \sin\theta)^{3/2}}{\theta^{1/2}} \frac{r^{5/2}}{2\sqrt{2}} \sqrt{S_0} C$$



$$\frac{d}{d\theta}\left[\frac{A^3}{P}\right]^{1/2} = 3P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$$

Let $x = (\theta - \sin\theta)^{3/2}\,\theta^{-0.5}$;

$$Q = \frac{x r^{5/2}}{2\sqrt{2}}\sqrt{S_0} C$$

Differentiating x w.r.t to θ and equating to zero.

$$\frac{dx}{d\theta} = \left[\frac{3}{2}(\theta - \sin\theta)^{1/2}(1-\cos\theta)(\theta)^{-0.5}\right] - \left[\frac{1}{2}\theta^{-3/2}(\theta - \sin\theta)^{3/2}\right] = 0$$

$$= \frac{3}{2}\frac{(\theta - \sin\theta)^{1/2}}{\theta}(1-\cos\theta) = \frac{1}{2}\left(\frac{\theta - \sin\theta}{\theta}\right)^{3/2}$$

$$3(1-\cos\theta) = \frac{\theta - \sin\theta}{\theta}$$

$$\sin\theta = \theta(3\cos\theta - 2);$$

$\theta = 308^\circ$ Radians.

Then the depth for maximum discharge.

$$y = r + r\cos\left(180^\circ - \frac{\theta}{2}\right) = r\left(1 + \cos 26^\circ\right) = 1.899r$$

$$308^\circ - 180^\circ = \frac{128^\circ}{2} = 64^\circ,$$

$$90^\circ - 64^\circ = 26^\circ$$

$$y = 0.95 d_o \quad \left[\because \frac{1.899}{2} = 0.95\right]$$

(a) Manning Equation - Maximum Discharge

$$\frac{d}{d\theta}\left[AR^{2/3}\right] = 0$$

$$\frac{d}{d\theta}\left[\frac{A^5}{P^2}\right]^{1/3} = 0$$

$$5P\frac{dA}{d\theta} - 2A\frac{dP}{d\theta} = 0$$

$$\frac{dA}{d\theta} = \frac{r^2}{2}(1-\cos\theta) \quad \left[\because A = \frac{r^2}{2}(\theta - \sin\theta)\right]$$

$$\frac{dP}{d\theta} = r \quad [\because P = r\theta]$$

$$\therefore\ 5(r\theta)\left(\frac{r^2}{2}\right)(1-\cos\theta)-2\frac{r^2}{2}(\theta-\sin\theta)r=0$$

$$5\frac{r^3}{2}\theta(1-\cos\theta)=r^3(\theta-\sin\theta)$$

$$\therefore\ \theta(1-\cos\theta)=\frac{1}{2.5}(\theta-\sin\theta)$$

$$5\theta(1-\cos\theta)=2(\theta-\sin\theta)$$

$$5\theta-5\theta\cos\theta=2\theta-2\sin\theta$$

$$3\theta=5\theta\cos\theta-2\sin\theta$$

$$\theta=302^\circ 22'$$

$$y=r-r\cos\frac{\theta}{2}=1.876r=1.876\frac{d_0}{2}$$

$$\therefore\ \boxed{y=0.938\ d_0}$$

(b) Circular section (Maximum velocity)

Using Manning equation

$$A=\frac{r^2}{2}(\theta-\sin\theta);R=\frac{r}{2}\left(1-\frac{\sin\theta}{\theta}\right)$$

$$V\propto R$$

$$\frac{d}{d\theta}\left[R^{2/3}\right]=0$$

$$\frac{d}{d\theta}\left[\frac{A^{2/3}}{P^{2/3}}\right]=0$$

$$P\frac{dA}{d\theta}-A\frac{dP}{d\theta}=0$$

$$(r\theta)\left[\frac{r^2}{2}(1-\cos\theta)\right]-\frac{r^2}{2}(\theta-\sin\theta)r=0$$

$$\therefore\frac{r^3}{2}\theta(1-\cos\theta)=\frac{r^3}{2}(\theta-\sin\theta)$$

$$\boxed{-\theta \cos\theta + \sin\theta = 0}$$

$$\theta = \tan\theta$$

$$\theta = 257^\circ 27' 56'' \approx 257^\circ 30'$$

The depth of water for maximum velocity is $y = r + r\cos\left(180 - \frac{257.5}{2}\right) = r + r\cos 51.25^\circ$ =

0.81 diameter = 0.81d_0

Problem

What would be the difference in discharge when it is running full and when it is

under $\frac{y_n}{d_0} = 0.938$

Solution

$\frac{y_n}{d_0} = 0.938$	$\frac{AR^{2/3}}{d_0^{\ 8/3}} = 0.3353$
$\frac{y_n}{d_0} = 1.0$	$\frac{AR^{2/3}}{d_0^{\ 8/3}} = 0.3117$

$$\frac{Q_{max}}{Q_{full}} = \frac{0.3353}{0.3117} = 1.0757$$

i.e. Maximum discharge is 7.6% higher than discharge in pipe when flowing full.

If Manning's equation is used.

If Chezy's equation is used, $\frac{y_n}{d_0} = 0.95$

$$\frac{A}{d_0^{\ 2}} = 0.77072$$

$$\frac{P}{d_0} = 2.69057$$

$$\frac{R}{d_0} = \frac{\dfrac{A}{d_0^{\,2}}}{\dfrac{P}{d_0}} = 0.28645$$

$$\therefore \frac{AR^{1/2}}{d_0^{\,5/2}} = 0.41249$$

When full $\dfrac{AR^{1/2}}{d_0^{\,5/2}} = \dfrac{\pi d_0^{\,2}}{4}\sqrt{\dfrac{d_0}{4}} = \dfrac{\pi}{4}\sqrt{\dfrac{1}{4}} = 0.39269$

$$\frac{Q_{max}}{Q_{full}} = \frac{0.41249}{0.39269} = 1.0504$$

$\therefore 5.04\%$ excess.