20.2 Normal and Critical Slopes

When discharge and roughness are given, the Manning formula can be used for determining the slope of the prismatic channel in which the flow is uniform at a given depth y_n. The slope thus determined is called Normal Slope S_n.

Now by changing this slope, the normal depth could be made equal to critical uniform flow for a given Q and n. This slope is called Critical slope S_c.

The smallest critical slope that sustains a given normal depth is called limiting slope S_L for a given shape and roughness.

By adjusting the slope and discharge if critical uniform flow is obtained that it is called Critical slope of normal depth S_{cn}.

These definitions will be illustrated in the following examples.

1. Normal and Critical Slopes:

Example: Rectangular open channel has a bottom width of 6.0 m, n = 0.02.

a) For y_n = 1.0 m, Q = 11 m^3/s, find normal slope.

b) Find the limiting critical slope and normal depth of flow for Q = 11 m^3/s.

c) Find the critical normal slope given y_n = 1.0 m and determine the discharge for this depth and slope.

Solution:

$$(a) \quad A = by = 6*1 = 6\ m^2, \quad P = b + 2y = 6 + 2*1 = 8m^2, \quad R = \frac{A}{P} = \frac{6}{8} = 0.75m$$

$$S_n = \frac{(Qn)^2}{A^2 R^{4/3}} = \left[\frac{11*0.02}{6*(0.75)^{2/3}}\right]^2 = 0.001972$$

$$\text{Froude } number = \frac{\overline{V}}{\sqrt{gy_n}} = \frac{Q}{A\sqrt{gy_n}} = \frac{11}{6\sqrt{9.81*1}} = 0.5853$$

$$\therefore \ \text{subcritical}\left(y_n > y_c\right). \quad \text{Hence mild slope.}$$

(b) For critical flow, $\frac{\overline{V}^2}{2g} = \frac{D}{2}$ $D = \frac{A}{T} = \frac{by}{b} = y$

$$\frac{\left[\frac{Q}{by_n}\right]^2}{2*9.81} = \frac{y_c}{2}$$ but $y_c = y_n$ for the uniform critical flow.

$$y_c = \left[\frac{11*11}{6*6*9.81}\right]^{1/3}$$ $y_c = 0.70\ m$

$A = 6*0.7 = 4.2\ m^2$ $P = 6 + 1.4 = 7.4$

$$R = \frac{4.2}{7.4} = 0.57\ m$$

Critical slope:

$$S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left[\frac{0.02*11}{4.2*(0.57)^{2/3}}\right]^2 = 0.0058$$

$S_c > S_0$

(c) If $y_n = 1.0$ $A = 6m^2$ $P = 8.0\ m,$ $R = 0.75$ $F = 1 = \frac{\overline{V}}{\sqrt{gy}}$

$\therefore \overline{V} = \sqrt{9.81*1} = 3.1\ m/s$

$$3.1 = \frac{1}{0.02}(0.75)^{2/3} S_{cn}^{1/2}$$

$$S_{cn} = \left[\frac{3.1*0.02}{(0.75)^{2/3}}\right]^2 = 0.00564$$

$Q = 3.1*6 = 18.6\ m^3/s$

Problem: A trapezoidal channel has a bottom width of 6 m, side slopes of 2: 1 (H: V) and n = 0.025.

(a) Determine the normal slope at a normal depth of 1.00 m and the discharge is 11 m^3/s.

(b) Determine the normal slope and corresponding normal depth when the discharge is 11 m^3/s.

(c) Determine the critical slope at the normal depth of 1.00 m and calculate the corresponding Q.

$$(a)\quad A = (b+2y)y = (6+2)1 = 8\ m^2$$

$$P = b+2y\sqrt{1+m^2} = 6+2\sqrt{5} = 10.472\ m$$

$$R = 0.7639\ m \qquad AR^{2/3} = 6.685$$

$$Q = \frac{1}{n}AR^{2/3}\ S_n^{1/2} \qquad S_n = \left[\frac{nQ}{AR^{2/3}}\right]^2 = 1.692*10^{-3}$$

$$S_n = 0.001692$$

$$(b)\quad S_c = ? \qquad F = 1 = \frac{V}{\sqrt{gD}}$$

$$V = \sqrt{gD} = \sqrt{9.81\ D}$$

$$D = \frac{y(6+2y)}{(6+2my)}, \qquad A = (6+2y)y$$

$$D = \frac{y(6+2y)}{(6+4y)}$$

$$\overline{V} = \frac{Q}{A} = \frac{11}{(6+2y)y} = \sqrt{9.81*\frac{y(6+2y)}{(6+4y)}}$$

$$11\sqrt{6+4y} = \left[(6+2y)y\right]^{3/2}\sqrt{g}$$

Squaring

$$121\ (6+4y) = g(6+2y)^3\ y^3$$

$$121\ (6+4y) = 9.81(6+2y)^3\ y^3$$

By trial and error

Say $y_c = 0.648$ m

$$A = (6+2(0.648))\ 0.648 = 4.7278\ m^2$$

$$P = b+2y\sqrt{5} = 8.8979\ m$$

$$R = 5.313*10^{-1} \qquad AR^{2/3} = 3.1016$$

$$S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = 0.007861$$

	R.H.S	L.H.S
y = 0.65	121 (6+2.6) = 1040.6	1048.039
y = 0.648	1039.63	1036.689

Say $y_c = 0.648$ m

$A = (6 + 2(0.648))\ 0.648 = 4.7278\ m^2$

$P = b + 2y\sqrt{5} = 8.8979$ m

$R = 5.313*10^{-1} \qquad AR^{2/3} = 3.1016$

$$S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = 0.007861$$

(c) *Given normal depth = 1 m*

$A = (6+2)1 = 8m^2$

$P = (6 + 2\sqrt{5}) = 10.472$ m

$R = 0.7639$ m

$T = b + 2my = 6 + 2*2*1 = 10$ m

$$D = \frac{A}{T} = 0.8$$

$V_c = \sqrt{gD} = \sqrt{9.81*0.8} = 2.801$ m / s

$$2.801 = \frac{1}{0.025}*(0.7639)^{2/3} S_{cn}^{1/2}$$

$$S_{cn}^{1/2} = \left[\frac{2.801*0.025}{(0.7639)^{2/3}}\right] = (8.3809*10^{-2})$$

$S_{cn}^{1/2} = 0.007024$

$Q = 2.801*8 = 22.408\ m^3/s$

Example:

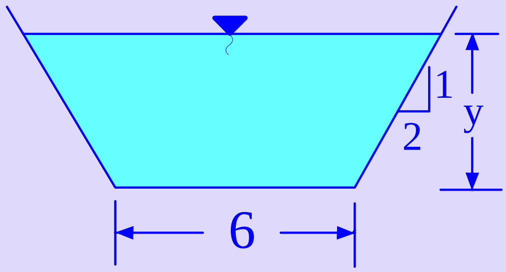
A trapezoidal channel has a bottom width of 6 m, side slopes of 2:1 and, n = 0.025.

(a) Determine the normal slope at a normal depth of 1.2 m when the discharge is 10 m^3/s.

(b) Determine the critical slope and the corresponding normal depth when the Q = 10 m^3/s.

(c) Determine the critical slope at y_n = 1.2 m and compute the corresponding Q.

Solution:



Trapezoidal

$A = 10.08\ m^2$ $\qquad \bar{V} = \frac{12}{10.08} = 1.1905\ m/s,$

$P\ =\ 6 + 2(1.2)\sqrt{5}\ =\ 11.36\ m$ $\qquad R = 0.8868$

$$\bar{V} = \frac{1}{n} R^{\frac{2}{3}}\ S_n^{\frac{1}{2}}$$

$$S_n = \frac{n^2 \bar{V}^2}{R^{4/3}} = \frac{(0.025)^2 (1.1905)^2}{(0.8868)^{4/3}}$$

$$S_n = 1.039 * 10^{-3} = 0.00104$$

(b) Critical depth $y_c = ?$

$$Z = \frac{Q}{\sqrt{g}} = A\sqrt{D} \qquad Z = \frac{12}{\sqrt{g}} = 3.8313$$

$$\bar{V}_n = \frac{Q}{A} = \frac{12}{5.0922} = 2.3565\ m/s$$

$$\therefore\ \ \bar{V}_c = \frac{1}{n} R^{2/3} S_n^{1/2} \qquad \therefore\ \ S_n = \frac{n^2 V_c^2}{R^{4/3}} = \left[\frac{0.025 * 2.3565}{R^{2/3}}\right]^2$$

$$S_n = 0.014718 \qquad y_c =\ 0.69$$

(c) $y_n = 1.2\ m \quad R\ =\ 0.8868 \quad A\ =\ 10.08\ m^2 \quad D = \frac{A}{T} = 0.9333\ m$

$$V_c = \sqrt{gD} = 3.0259\ m/s$$

Therefore the discharge $=$ Area $*$ Velocity $= 10.08\ *\ 3.0259 = 30.50\ m^3/s$

Solve by trial and error.

Graphical approach

Limit slope is the smallest critical slope for a given shape and roughness

(a) Determine S_c

$$Q = K\sqrt{S_c}$$

$$Q = \frac{1}{n} A R^{2/3} \sqrt{S_c}$$

$$Z_c = \frac{Q}{\sqrt{g}} \quad or \quad Q = Z_c\sqrt{g} = A\sqrt{D}\sqrt{g}$$

For rectangular channel: It can be written as

$$Q = \frac{1}{n} by \left(\frac{by}{b+y}\right)^{2/3} \sqrt{S_c}$$

also

$$Q = by\sqrt{\frac{by}{b}}\sqrt{g} = by^{1.5}\sqrt{g}$$

Rewriting the equation

$$y^{1.5} = \frac{Q}{b\sqrt{g}} \quad \text{or} \quad y = \left[\frac{Q}{b\sqrt{g}}\right]^{2/3}$$

Substituting the above value in Manning formula for discharge it may be written as

$$\therefore Q = \frac{1}{n} \frac{\left\{ b\left(\frac{Q}{b\sqrt{g}}\right)^{2/3} \right\}^{2/3}}{\left\{ b + 2\left(\frac{Q}{b\sqrt{g}}\right)^{2/3} \right\}^{2/3}} b\left(\frac{Q}{b\sqrt{g}}\right)^{2/3} \sqrt{S_c}$$

This is an Implicit function and solution is by trial and error approach.

Q					
S_c					

Alternatively

$$Q = \frac{b}{n}\left\{\frac{1}{b\sqrt{g}}\right\}^{2/3} Q^{2/3} \left[\frac{\frac{b}{\left(b\sqrt{g}\right)^{2/3}} Q^{2/3}}{b\left(b\sqrt{g}\right)^{2/3} + 2Q^{2/3}} \right]^{2/3} \sqrt{S_c}$$

If $\left(b\sqrt{g}\right)^{2/3} = C_1$

$$Q = \frac{b}{n}\frac{1}{C_1}\left[\frac{b}{C_1}\left\{\frac{1}{bC_1 + 2Q^{2/3}}\right\}\right]^{2/3} Q^{\frac{2}{3}*3}\sqrt{S_c}$$

$$\boxed{\frac{nC_1}{b\sqrt{S_c}} = \left(\frac{b}{C_1}\right)^{2/3}\left[\frac{1}{bC_1 + 2Q^{2/3}}\right]^{2/3} Q}$$

An equation in terms of S_c is obtained. So choose Q and obtain S_c, plot Q Vs S_c.

Example 3: Determine the limit slope of rectangular channel of 3 m width and roughness of 0.02. Consider the following cases

For depths (i) y = 0.5 m, (ii) y = 2 m.

Do we have limit slopes for these conditions?

Solution:

Section factor for critical flow

$$Z_c = \frac{Q}{\sqrt{g}} = A\sqrt{D} = A^{3/2}\ T^{-1/2} = by^{3/2} \qquad \text{for rectangular channel}$$

$$Q = b\,y^{3/2}\sqrt{g} = 3*\sqrt{9.81}*y^{3/2} = 9.3962\ y^{3/2}$$

But $Q = \dfrac{1}{n}AR^{2/3}S_c^{1/2}$

$$S_c = \frac{Q^2n^2}{A^2R^{4/3}} \qquad \therefore\ S_c = \frac{n^2\left[by^{3/2}\sqrt{g}\right]^2}{(by)^2\left[\dfrac{by}{b+2y}\right]^{4/3}}$$

Simplifying

$$\therefore\ S_c = gn^2y^{-1/3}\left[1+\frac{2y}{b}\right]^{4/3}$$

Case (i) $y_n = 2\text{m}$ $\qquad S_c = 9.6399 * 10^{-3}$

Note: There could be a situation where limit slope is not possible in expected range of flow depths.

Graphical approach:

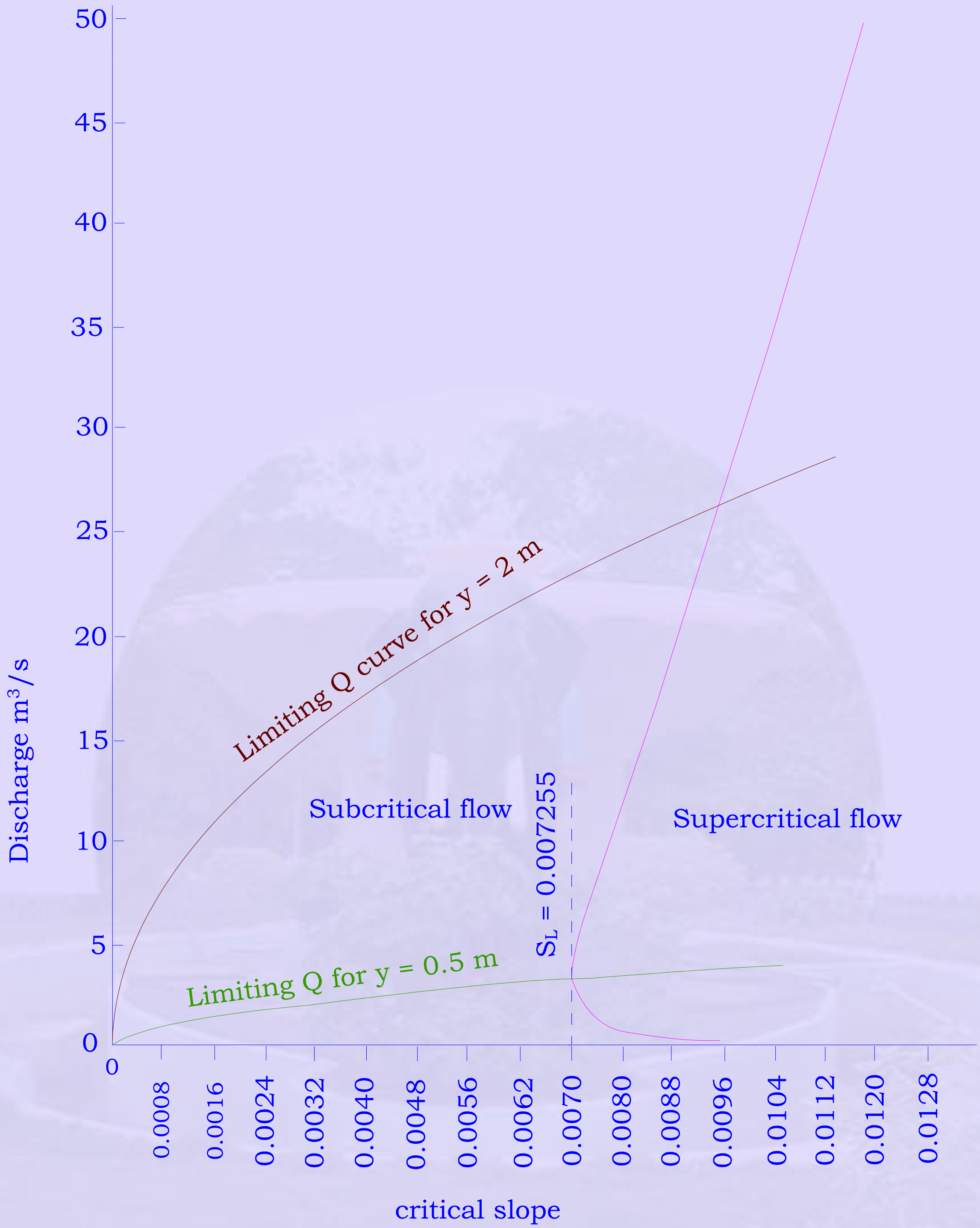
Table: To plot Q Vs S_c

y (m)	Q (m^3/s)	S_c
0	0	0
0.1	0.2971	0.008807
0.2	0.8405	0.007975
0.3	1.5440	0.007475
0.4	2.3770	0.007298

0.5	3.3322	0.007255
0.6	4.3670	0.0072866
0.7	5.5030	0.007364
0.8	6.7234	0.007474
0.9	8.0227	0.0076057
1.0	9.3963	0.007754
1.25	13.1317	0.008173
1.50	17.2621	0.008638
1.75	21.7527	0.009129
2.0	26.5767	0.009639
3.0	48.8245	0.011177
4.0	75.1702	0.01397

Limit slope computations:

Q (m^3/s)	y = 0.5 m, S_c	y = 2 m, S_c
1	$6.5741 * 10^{-4}$	$1.3646 * 10^{-5}$
2	$2.6296 * 10^{-4}$	$5.4585 * 10^{-5}$
3	$5.9166 * 10^{-3}$	$1.22817 * 10^{-4}$
4	0.0105185	$2.1834 * 10^{-4}$
5	0.01643	$3.4116 * 10^{-4}$
6	0.02367	$4.9124 * 10^{-4}$
8	0.04207	$6.6867 * 10^{-4}$
10	0.06574	$1.10535 * 10^{-3}$
15	0.147917	$3.0704 * 10^{--3}$
20	0.262963	$5.4585 * 10^{-3}$
30	0.59166	0.0122817
40	1.5018	0.2183
60	2.3667	0.04913
80	4.2074	0.08733
100	6.5741	



From analytical solutions:

$$\frac{y}{b} = \frac{0.5}{3.0} = \frac{1}{6}, \qquad S_L = 26.16\frac{n^2}{b^{1/3}} = \frac{26.16*(0.02)^2}{3^{1/3}} = 0.007255799$$

$$S_L = 0.007255, \; S_c = 9.0694*10^{-3} y^{-3}(3+2y)^{4/3}, Q = 3\sqrt{g}y^{3/2}$$

Analytical approach for obtaining limit slope:

Rectangular channel:

Consider a rectangular channel of width b and depth of flow y with Manning roughness coefficient n.

Then

$$V_c = \sqrt{g y_c} \qquad V_c = \frac{1}{n} R^{2/3} S_c^{1/2}$$

$$S_c = \frac{n^2 (b+2y)^{4/3}}{(by)^{4/3}} g y_c \qquad \therefore \; S_c = \frac{n^2 (b+2y)^{4/3}}{(by)^{4/3}} g y_c$$

$$S_c = n^2 g y_c \frac{(b+2y_c)^{4/3}}{b y_c}$$

By definition of limit slope,

$$\frac{dS_c}{dy} = \frac{d}{dy}\left\{ \frac{n^2 g y_c}{(b y_c)^{4/3}} (b+2y_c)^{4/3} \right\}$$

$$= \frac{d}{dy}\left\{ \frac{n^2 g}{b^{4/3}} \frac{(b+2y_c)^{4/3}}{y_c^{1/3}} \right\}$$

$$\frac{n^2 g}{b^{4/3}}\left\{ \frac{8}{3}(b+2y_c)^{\frac{4}{3}-1} y_c^{-\frac{1}{3}} + (b+2y_c)^{\frac{4}{3}}\left(-\frac{1}{3}\right) y_c^{-\frac{4}{3}} \right\} = 0$$

$$2*\frac{4}{3}(b+2y_c)^{\frac{1}{3}} y_c^{-\frac{1}{3}} = \frac{(b+2y_c)^{\frac{4}{3}}}{3} y_c^{-\frac{4}{3}}$$

$$2*4\frac{(b+2y_c)^{\frac{1}{3}}}{(b+2y_c)^{\frac{4}{3}}} = \frac{y_c^{-\frac{4}{3}}}{y_c^{-\frac{1}{3}}}$$

$$2*4\frac{\left(1+2\frac{y_c}{b}\right)^{\frac{1}{3}} b^{\frac{1}{3}}}{\left(1+2\frac{y_c}{b}\right)^{\frac{4}{3}} b^{\frac{4}{3}}} = y_c^{-1}$$

$$2*4\left[1+2\frac{y_c}{b}\right]^{-\frac{4}{3}+\frac{1}{3}} = b\, y_c^{-1}$$

$$2*4\left[1+2\frac{y_c}{b}\right]^{-1} = \frac{b}{y_c}$$

$$8 = \frac{b}{y_c}\left[1+2\frac{y_c}{b}\right]$$

$$8 = \frac{b}{y_c} + 2 \qquad \therefore \; 6 = \frac{b}{y_c}$$

S_c is maximum, when $\frac{y_c}{b} = \frac{1}{6}$

$\therefore$ Substituting into equation we can get the expression of limiting slope.

$$S_c = \frac{n^2 g y_c}{(b y_c)^{4/3}}\left[1+2\frac{y_c}{b}\right]^{4/3} b^{4/3}$$

$$S_c = \frac{n^2 g y_c}{b^{4/3} y_c^{4/3}}\left[1+2*\frac{1}{6}\right]^{4/3} b^{4/3}$$

$$= \frac{n^2 g}{b^{1/3}} \frac{1}{\left(\frac{y_c}{b}\right)^{1/3}}\left[1+\frac{1}{3}\right]^{4/3}$$

$$\boxed{S_L = \frac{n^2}{b^{1/3}}\left[9.81*\frac{6^{1/3}*4^{4/3}}{3^{4/3}}\right]^{4/3} = 26.157\frac{n^2}{b^{1/3}}}$$

$S_L = 26.157\frac{n^2}{b^{1/3}}$ or $2.67\frac{n^2 g}{b^{1/3}}$ in which b is in meter.

20.2.1 Froude Criteria for Sub Critical and Super Critical Flow

$$F = \frac{V}{\sqrt{gD}} = \frac{R^{2/3} S_0^{1/2}}{n\sqrt{gD}}$$

$$\left(S_0 < S_L \text{ sub critical, } S_0 > S_L \text{ super critical, } \frac{S_0}{S_L} = 1 \text{ critical} \right)$$

For rectangular channel.

$$A = by, \quad R = \frac{by}{b+2y}, \quad D = y$$

$$F = \frac{(by)^{2/3} \sqrt{S_0}}{n\sqrt{gy}\,(b+2y)^{2/3}}$$

If flow is critical uniform flow then

$$S_0 = \frac{n^2 gy (b+2y)^{4/3}}{(by)^{4/3}}$$

$$\frac{S_0}{S_L} = \frac{n^2 gy (b+2y)^{4/3} b^{1/3}}{2.67 n^2 g (by)^{4/3}}$$

$$\frac{S_0}{S_L} = \frac{g\left(1+\frac{2y}{b}\right)^{4/3} b^{4/3}\, b^{1/3}}{26.16\; y^{\frac{4}{3}-1}\, b^{4/3}}$$

$$\frac{S_0}{S_L} = \frac{9.81}{26.16} \frac{\left(1+\frac{2y}{b}\right)^{4/3} b^{1/3}}{y^{1/3}}$$

$$2.667 \frac{S_0}{S_L} = \frac{\left(1+\frac{2y}{b}\right)^{4/3}}{\left(\frac{y}{b}\right)^{1/3}}$$

$$\left(1+\frac{2y}{b}\right)^{4/3} = \left(\frac{y}{b}\right)^{1/3} 2.667 \frac{S_0}{S_L}$$

$$\left(1+\frac{2y}{b}\right)^{4} = \left(\frac{y}{b}\right)\left(2.667 \frac{S_0}{S_L}\right)^3$$

$$\left(1+\frac{2y}{b}\right)^{4} - \left(\frac{y}{b}\right)\left(2.667 \frac{S_0}{S_L}\right)^3 = 0$$

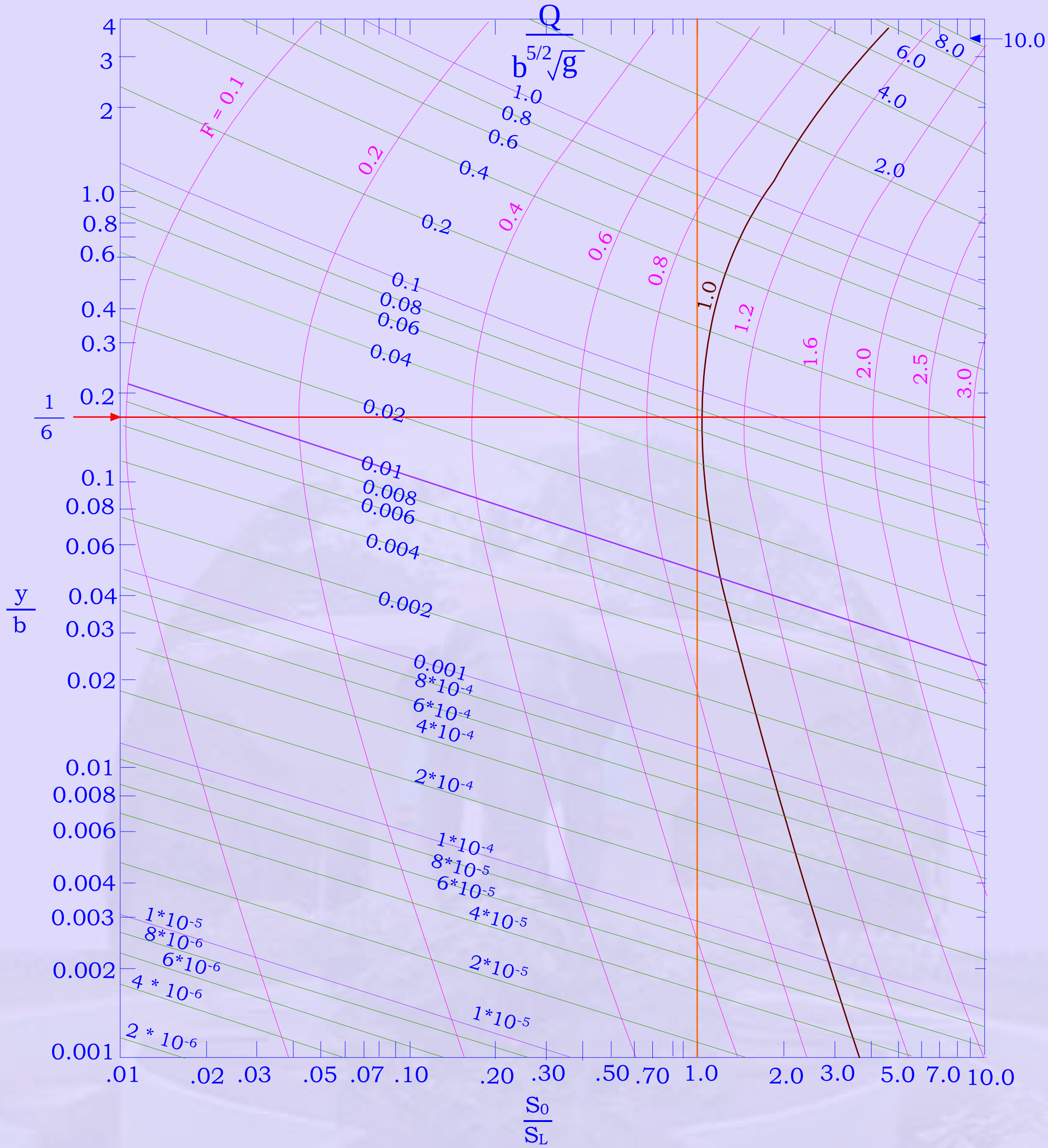
$$\left(\frac{b}{y}\right)\left(1+\frac{2y}{b}\right)^{4} - \left(2.667 \frac{S_0}{S_L}\right)^3 = 0$$

There are two solutions of $\frac{y_c}{b}$ for $\frac{S_0}{S_L} > 1$ and

one solution has a $\frac{y}{b} > \frac{1}{6}$. The other solution $\frac{y}{b} < \frac{1}{6}$.

The flow is super critical between these two values $\frac{y}{b}$ and it is sub critical for all the other values of $\frac{y}{b}$

If $\frac{S_0}{S_L} = 1; \ \frac{y}{b} = \frac{1}{6}$, Hence flow is critical and also the Froude number will be maximum *at* $\frac{y}{b} = \frac{1}{6}$.



With variation of $\frac{y}{b}$ with $\frac{S_0}{S_L}$ for uniform flow in rectangular channel

For $\frac{S_0}{S_L} < 1.0$ for all $\frac{y}{b}$ only sub critical exists.

For $\frac{S_0}{S_L} > 2.0$ the flow is super critical for most of the practical range of $\frac{y}{b}$

Maximum value of Froude number:

$$\text{Froude number } F = \frac{1}{n}\left(\frac{by}{b+2y}\right)^{2/3} \frac{\sqrt{S_0}}{\sqrt{gy}}$$

$$S_L = 26.16 \, \frac{n^2}{b^{1/3}}$$

$$F^2 = \frac{1}{n^2}\frac{b^{4/3}\left(y^{2/3}\right)^2}{b^{4/3}\left(1+\frac{2y}{b}\right)^{4/3}}\left\{\frac{\sqrt{S_0}}{\sqrt{gy}}\right\}^2$$

$$F^2 = \frac{1}{n^2}\frac{y^{4/3}}{\left(1+\frac{2y}{b}\right)^{4/3}}\frac{S_0}{S_L}$$

$$= \frac{1}{n^2 g}\frac{y^{1/3}}{\left(1+\frac{2y}{b}\right)^{4/3}}S_0$$

$$F^2 = \frac{26.16}{b^{1/3}S_L\ g}\frac{\left(y^{2/3}\right)}{\left(1+\frac{2y}{b}\right)^{4/3}}S_0$$

$$F = \sqrt{\frac{26.16}{9.81}}\left(\frac{S_0}{S_L}\right)^{1/2}\frac{\left(\frac{y}{b}\right)^{1/6}}{\left(1+\frac{2y}{b}\right)^{2/3}}$$

$$F = 1.632\left(\frac{S_0}{S_L}\right)^{1/2}\frac{\left(\frac{y}{b}\right)^{1/6}}{\left(1+\frac{2y}{b}\right)^{2/3}}$$

$\frac{dF}{dy} = 0$ condition for maximum and occurs at $\frac{y}{b} = \frac{1}{6}$

$$F_{max} = \sqrt{\frac{S_0}{S_L}}$$

Given n, S_0 channel width could be estimated such that the Froude number will never exceed a predetermined value irrespective of the value of discharge.

Problem:

Given $S_0 = 0.0025$, n = 0.25 estimate the width of the channel such that maximum Froude number is 0.5 irrespective of the discharge.

Sridharan and Lakshmana Rao have presented the design chart for rectangular channel and the details are as follows

$$S_L = 26.16\frac{n^2}{b^{1/3}}$$

$$Q = \frac{1}{n}AR^{2/3}\ S_0^{1/2}$$

$$Q = \frac{1}{n}\frac{(by)^{5/3}S_0^{1/2}}{(b+2y)^{4/3}}$$

$$\frac{Q}{b^{5/2}\sqrt{g}} = 1.632\left(\frac{S_0}{S_L}\right)^{1/2}\frac{\left(\frac{y}{b}\right)^{5/3}}{\left(1+\frac{2y}{b}\right)^{2/3}}$$

A design chart is created $\frac{y}{b}$ Vs $\frac{S_0}{S_L}$ for different values of $\frac{Q}{\sqrt{g}b^{5/2}}$ for different constant values of Froude number.

Reference:

1. Jones L.E. and Tripathy B.N. "Critical slopes for Trapezoidal channels ", ASCE HY1, 4202, Vol. 91, pp 85 - 91.

2. Nagar S. Lakshmana Rao and Kalambar Sridharan, "Limit slope in uniform flow computations", Proceedings ASCE Jl. Vol. 96, No. Hy1, Jan. 1970, p 7011, pp 95 to 102.

Problem:

1. Show that for Trapezoidal channel that there does not exist any limit slope for when

$$m \lesseqgtr 0.5$$

2. Show that the limit slope for trapezoidal channel is given by the following equation.

$$4m^2\sqrt{m^2+1}\left(\frac{y}{b}\right)^3 + \left(10m^2 - 4m\sqrt{m^2+1}\right)\left(\frac{y}{b}\right)^2 + \left(10m - 6\sqrt{m^2+1}\right)\frac{y}{b} + 1 = 0$$

in which m is the side slope.

3. Show that for circular channel the limit slope is given by

$$S_L = 33.06\frac{n^2}{d_0^{\frac{1}{3}}}$$

in which d_0 is the diameter of the circle in feet and the subtended angle by the free surface at the centre corresponds to $132^\circ 06'$

4. Establish that for triangular channel the limit slope will be zero.

5. For trapezoidal channel, show that

$$\text{(i) } S_x = \left(\frac{S_0 b^{1/3}}{F^2 g n^2}\right) = \frac{\left[1 + 2\sqrt{m^2+1}\left(\frac{y}{b}\right)\right]^{4/3}}{\left(1+2m\frac{y}{b}\right)\left(1+m\frac{y}{b}\right)^{1/3}\left(\frac{y}{b}\right)^{1/3}}$$

$$\therefore S_* = f(m, \eta) \text{ in which } \eta = \frac{y}{b}.$$

$$\text{(ii) } S_*^3 = \frac{(1+2\eta)\left(\sqrt{1+m^2}\right)^4}{(1+2m\eta)^3(1+2m\eta)^{\eta}}$$

a fifth degree equation in η $[if\ m = 0,\ \text{then 4 th degree}]$

Five roots: atleast one +ve real root, two roots are imaginary.

$$S_L = \frac{dS_*}{dy} = 0,$$

$$8\eta\left(1+m^2\right)(1+m\eta)(1+2m\eta) - \left(1+2\eta\sqrt{1+m^2}\right)\left(1+10m\eta+10m^2\eta^2\right) = 0$$

Example:

A Trapezoidal channel with a bottom of 6.2 m and side slope of 0:5; 1, n = 0.02 develop a graph Q Vs S_c and obtain the limiting critical slope.

Hint:

$$\text{Critical flow } \frac{\overline{V}^2}{2g} = \frac{D}{2}$$

$$A = (6.2+0.5y)\,y$$

$$P = (6.2+2*0.5y)\,y$$

$$R = \frac{(6.2+0.5y)\,y}{(6.2+2*0.5y)\,y}$$

$$\overline{V} = \frac{Q}{(6.2+0.5y)\,y}$$

$$\frac{Q^2}{(6.2+0.5y)^2 y^2} * \frac{1}{2g} = \frac{(6.2+0.5y)\,y}{2(6.2+y)}$$

$$\frac{Q^2}{g} = \frac{(6.2+0.5y)^3 y_c^3\, g}{(6.2+y_c)}$$

$$S_{Cn} = \frac{(6.2+0.5y)^3 y_c^3\, g}{(6.2+y_c)[6.2+0.5y_c]^2 y_c^2 \left[\dfrac{(6.2+0.5y_c)}{(6.2+2*0.5y_c)}\right]^{4/3}}$$

Select different values of y_c and calculate S_{Cn} and Q