23.3 Dynamic Equation for Steady Gradually Varied Flow

Consider the flow profile in an elementary length dx of an open channel as shown in Figure 23.1.

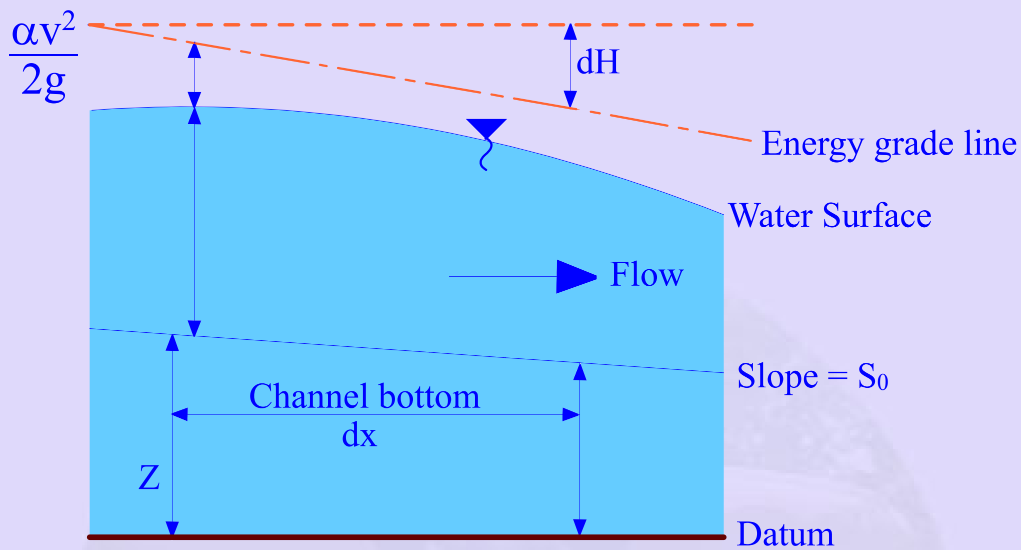

Figure: 23.1 Derivation of the gradually varied flow equation

The total head above the datum at a section is

$$H = z + y + \frac{\alpha v^2}{2g} \quad (23.1)$$

where H is the total head; z is the elevation of the channel bottom; y is the depth of flow; α is the energy coefficient; g is the acceleration due to gravity; and V is the average velocity of flow through section. Here, bottom of the channel is considered on the X-axis. Equation (23.1) is differentiated with respect to x to obtain.

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \alpha \frac{d}{dx}\left(\frac{V^2}{2g}\right) \quad (23.2)$$

As the slope of the channel bottom is assumed small, $\text{Sin}\,\theta \approx \tan\theta \approx \theta$, in which θ is the angle of the channel bottom with horizontal. Slope is considered positive if it depends in the direction of flow. Therefore, referring to Figure 23.1, slope of the energy grade line, $S_f = -\frac{dH}{dx}$, and slope of the channel bottom, $S_0 = -\frac{dz}{dx}$.

Equation (23.2) becomes

$$\frac{dy}{dx} + \alpha \frac{d}{dx}\left(\frac{V^2}{2g}\right) = S_0 - S_f \quad (23.3)$$

Velocity, V can be expressed in terms of the flow rate, Q and area of the cross section, A.

$$V=\frac{Q}{A} \quad (23.4)$$

Noting that flow rate, Q remains constant with respect to x (no lateral inflow or outflow), but area, A changes, differentiating Equation (23.4) with respect to x and subsequent substitution in Equation (23.3) leads to

$$\frac{dy}{dx}+\frac{\alpha Q^2}{2g}\left(\frac{-2}{A^3}\right)\frac{dA}{dx} \quad (23.5)$$

However, for a prismatic channel

$$\frac{dA}{dx}=\frac{dA}{dy}\frac{dy}{dx}=T\frac{dy}{dx} \quad (23.6)$$

where, T = free surface width. Substitution of Equations (23.4) and (23.6) in Equation (23.5) and subsequent simplification results in the following gradually varied flow equation,

$$\frac{dy}{dx}=\frac{S_o-S_f}{1-\frac{\alpha Q^2 T}{gA^3}} \quad (23.7)$$

Equation (23.7) is a non-linear first-order differential equation. In this equation, slope of the energy grade line, S_f may be estimated using the Manning's equation.

$$S_f=\frac{n^2Q^2}{A^2R^{4/3}} \quad (23.8)$$

where n is the Manning roughness coefficient; and R is the hydraulic mean radius.