25.1 Computation of Gradually Varied Flow

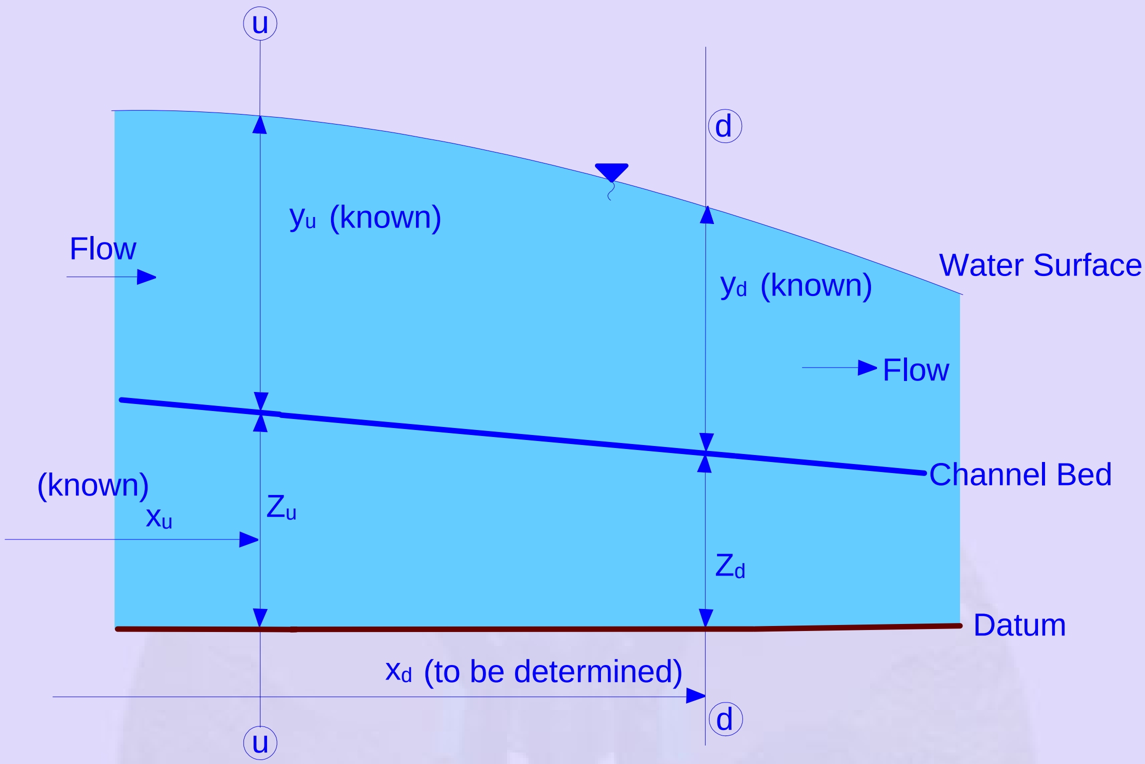
Qualitative sketching of water surface profiles in channels having gradually varied flow has been discussed elsewhere. However, quantitative information on the variation of the flow depth and flow velocity along a channel is required in many engineering applications. For example, construction of a dam across a river raises water levels on the upstream side of the dam. Estimation of the extent of inundation in such a case is possible only by performing computations to determine the flow depths. Impounding of water behind a dam also changes the self cleansing ability of the river to assimilate the municipal waste discharged into it. Thus quantitative knowledge of flow depths and velocities is essential while conducting the Environmental Impact Assessment (EIA) studies also. These computations, generally known as GVF (gradually varied flow) computations determine the water-surface elevations along the channel length for specified (i) discharge, (ii) flow depth at any one location, (iii) the Manning roughness coefficient, (iv) longitudinal profile of the channel, and (v) channel cross-sectional parameters. Generally, systematic numerical procedures are used for this purpose. All these numerical procedures are either based on the numerical solution of the non-linear first - order ordinary differential equation for GVF (Eq. 23.7)

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{\alpha Q^2 T}{g A^3}} \qquad (23.7)$$

or on the direct application of the algebraic energy equation, using certain approximations. These methods for the GVF computation are presented in the following sections.

25.2 Direct Step method

In the Direct Step method, the location where the specified depth, y_d occurs is determined, given the location for the occurrence of depth, y_u. Consider the channel shown in figure 25.1. In this channel, say depth y_u occurs at a distance x_u from the reference point. Discharge, Q, channel bottom slope, S_0, the roughness coefficient, n and cross - sectional shape parameters (which relate A, P and R to y) are also known. The problem now is to determine the location x_d (fig. 25.1).



25.1 Definition sketch for Direct Step Method

Energy equation between sections u and d can be written as follows

$$z_u + y_u + \frac{\alpha_u V_u^2}{2g} = z_d + y_d + \frac{\alpha_d V_d^2}{2g} + \bar{S}_f (x_d - x_u) \qquad 25.1$$

where subscripts "u" and "d" denote the values at the corresponding sections, and $\bar{S}_f$ is the average slope of the energy grade line between sections u and d. It may be noted that the slope of the energy grade line, S_f can be determined using Equation 23.8.

$$S_f = \frac{n^2 Q^2}{A^2 R^{4/3}} \qquad (23.8)$$

S_f varies between sections u and d since the flow depth, and consequently A and R vary between these two sections. S_f may also due to variation in the roughness between the two sections. Following equations may be used to determine $\overline{S_f}$.

Arithmetic mean

$$\overline{S_f} = \frac{1}{2}\left(S_{f_u} + S_{f_d}\right) \qquad (25.2\ a)$$

Geometric mean

$$\overline{S_f} = \sqrt{\left(S_{f_u} * S_{f_d}\right)} \qquad (25.2\ b)$$

Harmonic mean

$$\overline{S_f} = \frac{2S_{f_u}S_{f_d}}{S_{f_u} + S_{f_d}} \qquad (25.2\ c)$$

Experience has indicated that the arithmetic mean (Eq. 25.2 a) gives the lowest maximum error, although it is not always the smallest error. Also, it is the simplest of the three approximations. Therefore, its use is generally recommended, and is used herein. Noting that the bed elevations Z_u and Z_d are related through the bed slope, S_0 and the distance between the sections, $(x_d - x_u)$, Eq. 25.1 can be written as

$$-\left(y_d + \alpha_d \frac{V_d^2}{2g}\right) + \left(y_u + \alpha_u \frac{V_u^2}{2g}\right) = \bar{S}_f\left(x_d - x_u\right) - S_0\left(x_d - x_u\right) \qquad (25.3)$$

However,

$$y_u + \alpha_u \frac{V_u^2}{2g} = y_u + \frac{\alpha_u Q^2}{2gA_u^2} = E_u$$

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$$y_d + \alpha_d \frac{V_d^2}{2g} = y_d + \frac{\alpha_d Q^2}{2gA_d^2} = E_d$$

where E_u and E_d are specific energies at section u and d, respectively. Equation 25.3 can now be used to determine x_d.

$$x_d = x_u + \frac{E_d - E_u}{S_0 - \frac{1}{2}\left(S_{f_u} + S_{f_d}\right)} \qquad 25.5$$

In equation 25.5, specific energies E_u *and* E_d, and the friction slopes, S_{f_u} and S_{f_d}, can be computed using the known values of (i) flow depths y_u *and* y_d, (ii) the flow rate Q, (iii) the roughness coefficient, n and (iv) the cross sectional shape parameters. Therefore, x_d can be computed easily. For example, for a wide rectangular channel (assuming $\alpha_u = \alpha_d = 1.0$).

$$E_u = y_u + \frac{q^2}{2gy_u^2}$$

$$(25.6)$$

$$E_d = y_d + \frac{q^2}{2gy_d^2}$$

and

$$S_{f_u} = \frac{n^2 q^2}{y_u^{10/3}}$$

$$(25.7)$$

$$S_{f_d} = \frac{n^2 q^2}{y_d^{10/3}}$$

Substitution *of Eqs (25.6) and (25.7) in Equation 25.5 yields*

$$x_d = \frac{x_u + (y_d - y_u) + \frac{q^2}{2g}\left[\frac{1}{y_d^2} - \frac{1}{y_u^2}\right]}{S_0 - \frac{q^2 n^2}{2}\left[\frac{1}{y_d^{10/3}} + \frac{1}{y_u^{10/3}}\right]} \qquad (25.8)$$

In Equation 25.8, y_u, y_d, S_0, q, n *and* x_u are known, and x_d can be determined easily. Now that the location of section d is known, it is used as the starting value for the next step. The water surface profile in the entire channel may be computed by increasing or decreasing the flow depth, and determining the locations where these depths occur. For example, say one is interested in determining the changes in flow depths in a mildly sloping river due to the construction of a dam. The flow depth just behind the dam, y_{dam} is known for the specified discharge, Q, the spillway length and the spillway configuration. Flow depth far upstream of the dam is equal to the normal depth, y_n since uniform flow conditions exist there, assuming that the channel is prismatic. By varying the flow depth value between y_{dam} and y_n in a systematic stepwise manner, and applying Eq. (25.5) recursively, the extent to which the dam affects the water levels can be determined. This is illustrated in example 25.1.

Direct step method has the following disadvantages:

- Interpolations become necessary if the flow depths are required at specified locations.
- It is inconvenient to apply this method to non prismatic channels because the cross-sectional shape at the unknown location should be known a priori.