26.2 Example 26.1

A wide rectangular channel having a bottom slope of 0.001 is carrying a flow of 3 $m^3/s/m$. A control structure is built at the downstream end which raises the water depth at the downstream end to 4.5 m. Determine the flow depth at a distance of 1000 m upstream of the control structure. Manning n for the channel is 0.012.

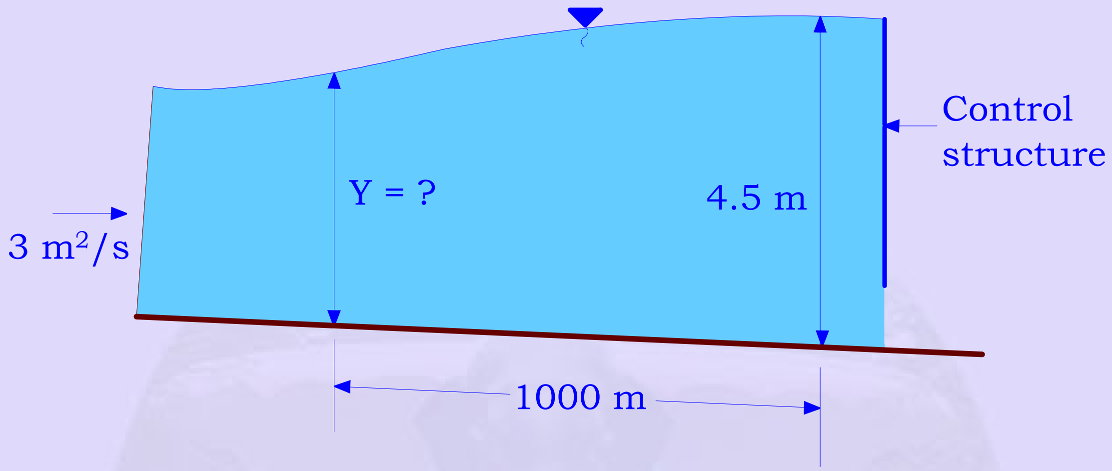

Fig. 26.2: Definition sketch for Example 26.1

Solution

- Divide the distance into two reaches as shown in Fig. 26.3. Section 2 is 500 m upstream of section 3, while section 1 is 500 m upstream of section 2. Flow depths Y_1 and Y_2 are determined as follows.

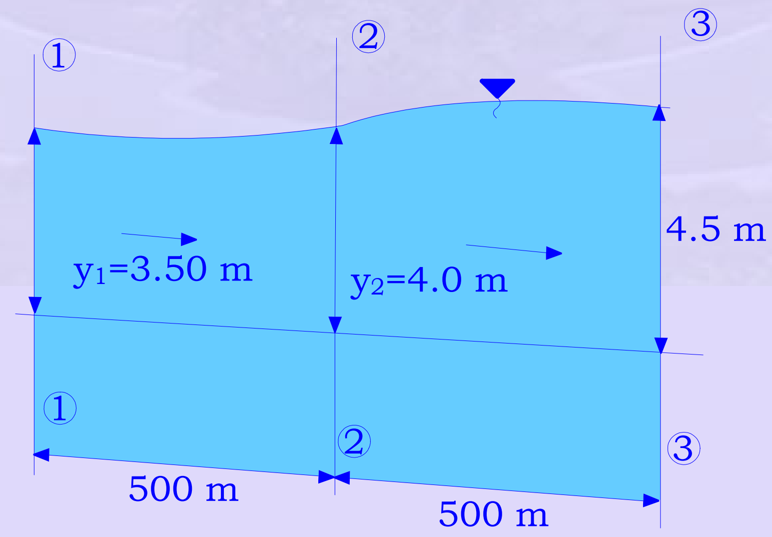

Fig. 26.3: Solution for Example 26.1

- Apply Eq.(26.2) between sections 2 and 3. Here,

  $\left(X_d - X_u\right) = 500\ m,\ S_0 = 0.001,\ n = 0.012,\ q = 3.0\ m^2/s\ \text{ and } y_3 = 4.5\ m.$ Y_2 is taken as upstream depth while Y_3 is taken as downstream depth. In this example, we know Y_d and we are determining Y_u by solving Eq. (26.2). Equation (26.2) simplifies to the following:

$$4.025 = y_2 + \frac{0.459}{y_2^2} - \frac{0.324}{y_2^{3.333}}$$

For different values of Y_2, R.H.S. of the above equation is as shown below

y_2 (m)	R.H.S (m)
3.99	4.0156
3.90	3.9267
3.999	4.0245
4.000	4.0255

Therefore y_2 = 4.000 m.

- Apply Eq. (26.2) between sections 2 and 3. Here,

  $\left(X_d - X_u\right) = 500\ \text{m},\ S_0 = 0.001,\ n = 0.012,\ q = 3.0\ m^2/s\ \text{ and } y_2 = 4.0\ m.$ Y_1 is taken as upstream depth while Y_2 is taken as downstream depth. Here, we know Y_d and we are determining Y_u. Equation (26.2) simplifies to the following:

$$3.532 = y_1 + \frac{0.459}{y_1^2} - \frac{0.324}{y_1^{3.333}}$$

For different values of Y_1, R.H.S of the above equation is as shown below,

y_1 (m)	R.H.S (m)
3.49	3.5227
3.50	3.5325
3.499	3.5315

Therefore, y_1 = 3.50 m.

- Either Eq. (26.1) for any general prismatic channel or Eq. (26.2) for a wide channel can be applied in the above step by step manner to determine the flow depth at any given location upstream of the control structure. Thus the entire water surface profile behind the control structure can be determined.

- In this example, a trial and error method is used for solving the non-linear algebraic equation. One can use Newton-Raphson method for the same purpose.
- In this, as well as in the example 25.1, more accurate results for water surface profile can be obtained by performing computations over more number of reaches for the same distance.

Example 26.2

A rectangular channel of 6.0 m width carries a discharge of 12.0 m^3/s. The channel slope is 0.0001 and the Manning's n = 0.018. There is a free over fall at the downstream end of the channel. Determine the flow depth at a section 500 m upstream of the free over fall. Use Standard Step method and one reach.

Solution

Unit discharge, q = 12.0 / 6.0 = 2 m^2/s

Critical depth, $y_c = \left(\frac{q^2}{g}\right)^{1/3} = 0.742 \text{ m}$

Normal depth, y_n:

$$q = \frac{1}{n} y_n \left(\frac{By_n}{B+2y_n}\right)^{2/3} \sqrt{S_0}$$

B = 6.0 m

S_0 = 0.0001

n = 0.018

$$y_n \left(\frac{6y_n}{6+2y_n}\right)^{2/3} = 3.6$$

Solving by trial and error; y_n = 2.8 m

$y_n > y_c$; therefore slope is MILD

Therefore, Critical depth occurs at the downstream end. Denoting section-2 as the section at the downstream end, section-1 on the section at 500 m upstream of the free over fall,

$$Z_1 + y_1 + \frac{q^2}{2gy_1^2} = Z_2 + y_2 + \frac{q^2}{2gy_2^2} + \frac{Q^2 n^2 \Delta x}{2}\left[\frac{1}{A_1^2 R_1^{4/3}} + \frac{1}{A_2^2 R_2^{4/3}}\right]$$

$$y_2 = 0.742 \text{ m}$$

$$Z_2 - Z_1 = -S_0 \Delta x = -0.0001 * 500 = -0.05$$

$$y_1 + \frac{0.2038}{y_1^2} - \frac{0.324}{y_1^2 (R_1)^{4/3}} = 2.238$$

$$R_1 = \frac{6y_1}{6 + 2y_1}$$

Solving by trial and error,

y_1 = depth of flow = 2.25 m

Example 26.3

Solve the problem in Example 26.2 using two reaches

Solution

Section – 1: Located at 500 m upstream of free over fall

Section – 2: Located at 250 m upstream of free over fall

Section – 3: Located at downstream end.

Consider sections 2 and 3

$$Z_2 + y_2 + \frac{q^2}{2gy_2^2} = Z_3 + y_3 + \frac{q^2}{2gy_3^2} + \frac{\Delta x\ Q^2 n^2}{2}\left[\frac{1}{A_3^2 R_3^{4/3}} + \frac{1}{A_2^2 R_2^{4/3}}\right]$$

$$y_3 = 0.742\,\text{M},\ \Delta X = 250 \text{ m}$$

$$Z_3 - Z_2 = -0.0001 * 250 = -0.025$$

With the above values, equation for y_2 is given as

$$y_2 + \frac{0.2038}{y_2^2} - \frac{0.162}{y_2^2 (R_2)^{4/3}} = 1.675 \text{ m};\ R_2 = \frac{6y_2}{6 + 2y_2}$$

Solving by trial and error,

y_2 = 1.655 m

Consider Sections 1 and 2

$$Z_1 + y_1 + \frac{q^2}{2gy_1^2} = Z_2 + y_2 + \frac{q^2}{2gy_2^2} + \frac{\Delta x\ Q^2 n^2}{2}\left[\frac{1}{A_2^2 R_2^{4/3}} + \frac{1}{A_1^2 R_1^{4/3}}\right]$$

$$y_2 = 1.655 \text{ m}$$

$$\Delta x = 250 \text{ m}$$

$$Z_2 - Z_1 = -0.0001 * 250 = -0.025$$

With the above values, Equation for y_1 is given as

$$y_1 + \frac{0.2038}{y_1^2} - \frac{0.162}{y_1^2 (R_1)^{4/3}} = 1.759$$

$$R_1 = \frac{6y_1}{6 + 2y_1}$$

Solving by trial and error,

y_1 = 1.740 m

Flow depth at 500 m upstream of the d/s end = 1.740 m. The above value is very much different from the value of 2.25 m obtained when only one reach is considered. This illustrates the importance of choosing a small value of Δx to obtain accurate results. However, Computational effort increases if a very small value of Δx is chosen. Above points should be kept in mind while performing GVF computations.