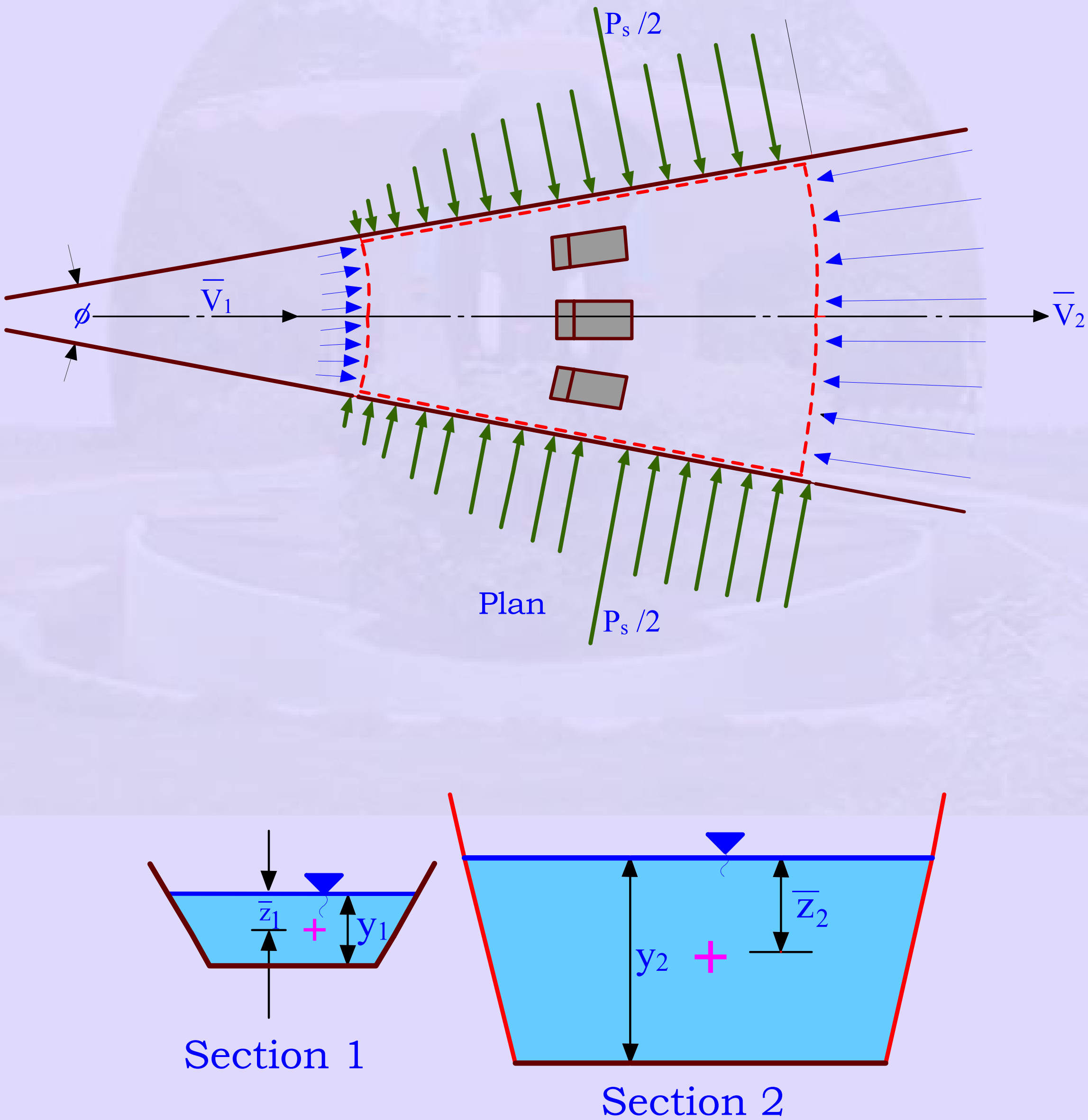
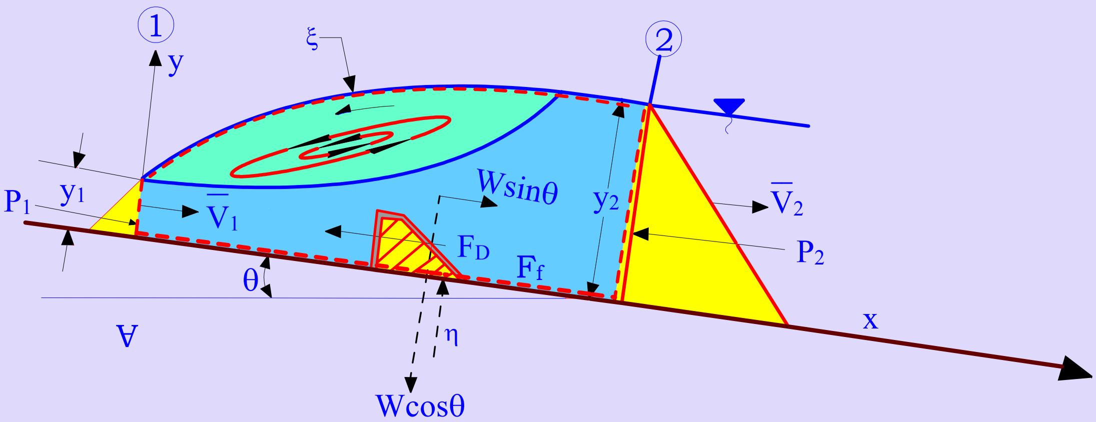
28.4 General Hydraulic Jump Equation

The internal equation of motion for a hydraulic jump can be integrated in two ways whether the control volume is defined on a microscopic scale. The hydraulic jump takes place over a short distance (of the order of five times the sequent depth) the transition is dominated by initial momentum flux and pressure forces due to sequent depth. Boundary shear forces are secondary in nature. Consider the situation shown in Figure 28.1 (unsubmerged, forced hydraulic jump in a radially diverging sloping channel). The macroscopic approach is as follows:





Longitudinal Section

Figure 28.1 - Schematic diagram of a hydraulic jump in free surface flows

Using the Green's theorem, the Reynolds equation for turbulent flow can be integrated over the control volume $\forall$ to obtain.

$$\underbrace{\int_\xi \rho\, \overline{u_i}\,\overline{u_j}\,\frac{\partial x_i}{\partial \eta}d\xi}_{(1)} + \underbrace{\int_\xi \rho\, \overline{u_i u_j}\,\frac{\partial x_j}{\partial \eta}d\xi}_{(2)} = \underbrace{-\int_\xi \bar{p}\,\frac{\partial x_i}{\partial \eta}d\xi}_{(3)} + \underbrace{\int \rho\, x_i\, d\forall}_{(4)} + \underbrace{\int_\xi \mu\,\frac{\partial \bar{u}_i}{\partial x_j}\frac{\partial x_j}{\partial \eta}d\xi}_{(5)} \qquad (28.1)$$

in which η is outwardly directed normal.

The first term represents the net flux of momentum through the boundary ξ, due to mean flow. The second term represents the net momentum transfer through the boundary due to ξ turbulence. The third term represents the pressure force (resultant mean normal) exerted on the fluid boundary ξ. The fourth term represents the net weight of the fluid within the control volume $\forall$ and the fifth term represents mean tangential force exerted on the boundary ξ.

A macroscopic momentum equation is obtained if the above equation is applied to the control volume $\forall$, shown in Fig 1.

Consider the momentum in the x direction, then it can be written as

$$\left\{\beta_2 \rho \overline{V}_2 Q - \beta_1 \rho \overline{V}_1 Q\right\} + \left(\rho I_2 - \rho I_1\right) = P_1 - P_2 + P_s \sin\left(\frac{\varphi}{2}\right) - F_D - W\sin\theta - F_f \qquad (28.2)$$

in which, pressure force, is force on the side wall.

$\beta = \int \overline{v}^2 \, dA/(QV),\ I = \int v' \, dA,\ P = \beta' g\rho A\overline{z}\cos\theta$, pressure force P_s , is force on the side wall.

The continuity equation is $Q = \overline{V}_1 A_1 = \overline{V}_2 A_2$ (28.3)

Equations 2 and 3 are to be solved simultaneously to determine the sequent depth, velocity $(v_{\hat{2}}, y_{\hat{2}})$ for given initial condition (v_1, y_1). If $S_0 = 0$, rectangular channel without baffles, and no side thrust, then it simplifies to the standard format (equation 4)

$$\frac{\overline{V}_1^2 y_1^2}{g} + \frac{y_1^2}{2} = \frac{\overline{V}_2^2 y_2^2}{g} + \frac{y_2^2}{2} \quad (28.4)$$

$$y_1 \overline{V}_1 = y_2 \overline{V}_2 \quad (28.5)$$

When solved results in

$$y_0^3 - \left(2F_1^2 + 1\right) y_0 + 2F_1^2 = 0 \text{ or } y_0 = \frac{1}{2}\left(\sqrt{1 + 8F_1^2} - 1\right) \quad (28.6)$$

in which y_0 is the sequent depth ratio $\frac{y_2}{y_1}$.

Bed friction decreases the ratio by about 4% at $F_1 = 10.0$. It is to be noted that the macroscopic approach yields only sequent depth ratio and no information regarding surface profile or the length of the jump. In radial stilling basins, sloping basins, forced hydraulic jump even the sequent depth ratio depends on the internal flow and hence the physical model is used for determining.