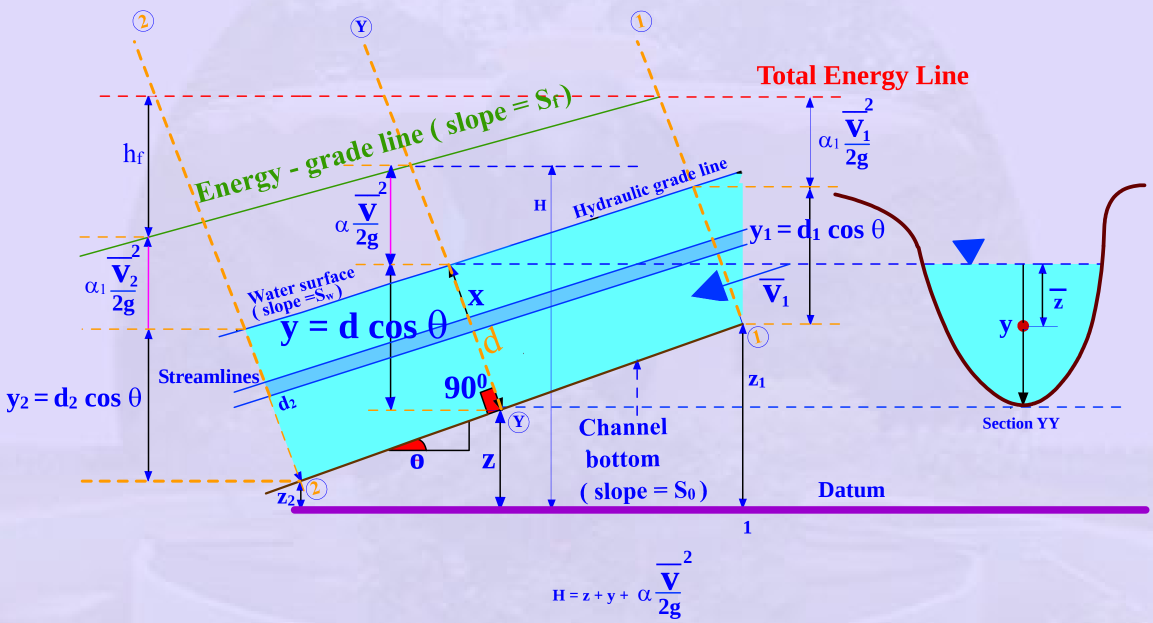
3.2 ENERGY IN FREE SURFACE FLOW

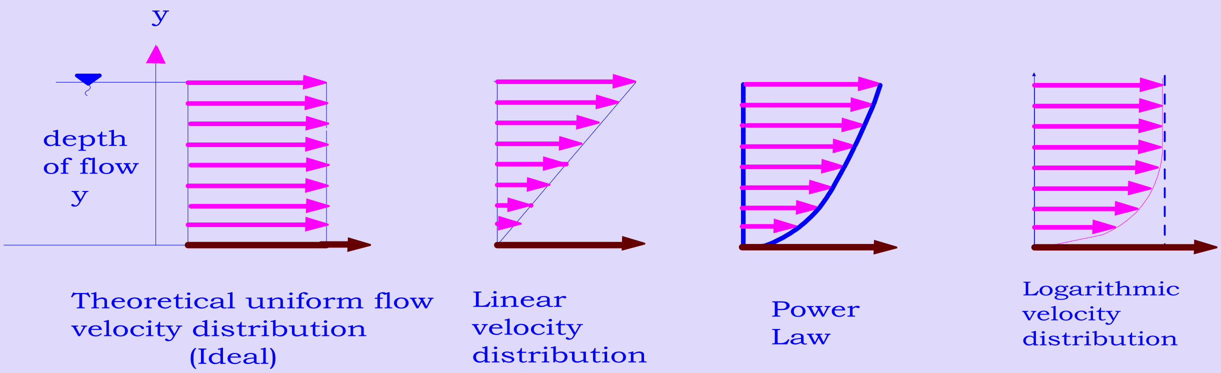
It is known in basic fluid mechanics that the total energy in (Newton-meter per Newton) of water along any streamline passing through a channel section may be expressed as the total head in meter of water, which is equal to the sum of the elevation (above a datum), the pressure head, and the velocity head. For example, with respect to the datum plane, the total head H at a section containing point X on a streamline of flow in a channel of large slope may be written as

$$H_x = d_x \cos\theta + \alpha \frac{\overline{V}_x^2}{2g} + z_x$$

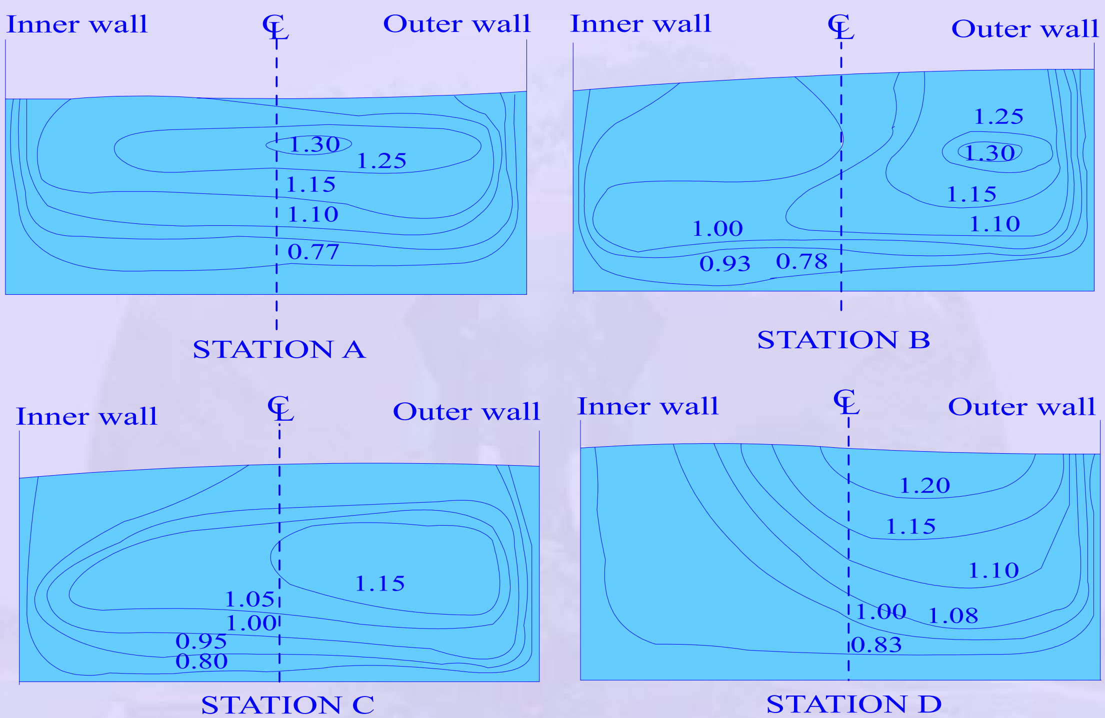


Energy in gradually varied open channel flow

in which z is the elevation of point Y above the datum plane, d is the depth of flow normal to the bed, y is the vertical depth below the water surface measured at the channel section, θ is the angle of the channel bottom with horizontal and $\frac{\overline{V}^2}{2g}$ is the mean velocity head of the flow in the streamline passing through point X. In view of the variation in velocity over the depth, the velocity head would be differing. The mean velocity obtained by integrating the velocity distribution is considered for the entire section $\overline{V} = \int_0^A v\,dA$. In order to account for the variation of the velocity due to non uniform pattern of velocity distribution, an energy correction factor α is used.

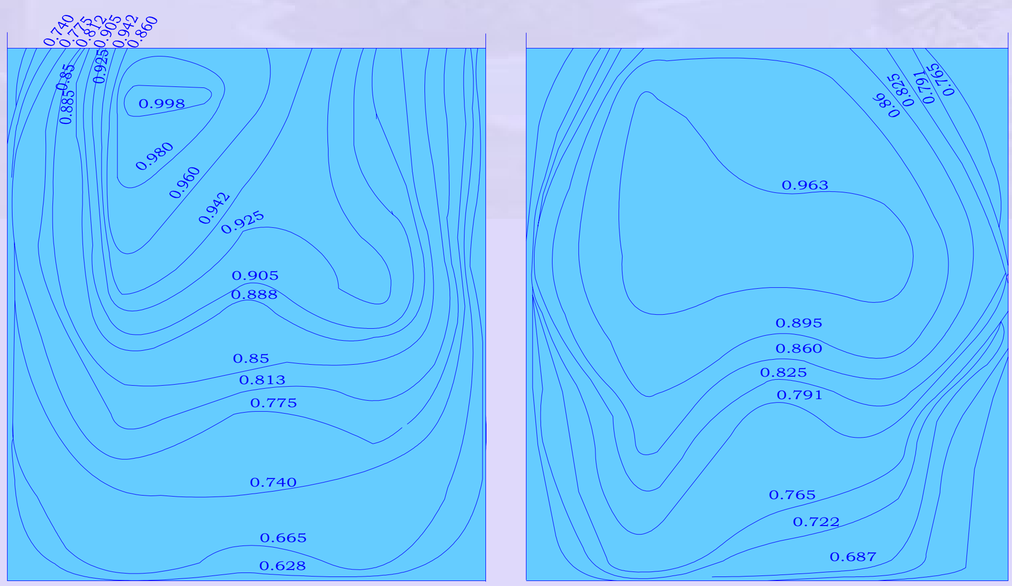


Typical velocity distribution



ISOVELS in open channel bend [Normalised with V_{max}]

Q = 83.5 lps, F = 0.41, R_e = 103460

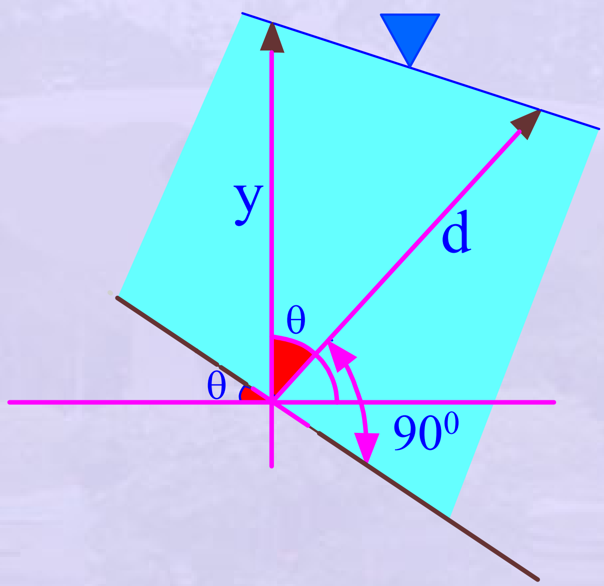


Section A
Q = 33.61 l/s, F = 0.2457

Section 4
R_e = 179574, n = 0.009834

Non-Dimensionalised isovels (v/v_{max})

In general, every streamline passing through a channel section will have a different velocity head, because of the non- uniform velocity distribution in actual flow. Only in an ideal parallel flow of uniform velocity distribution, can the velocity head be truly identical for all points on the cross section. In the case of gradually varied flow, however, it may be assumed, for practical purposes, that the velocity head for all points in a channel section are equal, and the energy coefficient (α) may be used for correcting for the over-all effect of the non-uniform velocity distribution. Thus, the total energy at the channel section may be written as $H = d\cos\theta + \alpha\frac{\overline{V}^2}{2g} + z$.



Normal and Vertical depths

for channel of small slope, $\theta \approx 0$ thus, the total energy at the channel section is $H = y + \alpha\frac{\overline{V}^2}{2g} + z$.

The slope of the energy line is denoted by S_f, the slope of water surface is denoted by S_w and, the slope of the channel bottom by $S_o = \sin\theta$ with an assumption that $\frac{\sin\theta}{\theta} \approx \frac{\tan\theta}{\theta} = 1$ (See box).

If the value of θ is taken as

(i) 6°; $\sin\theta = 0.1045$, $\tan\theta = 0.1051$

the difference is 0.0006 then $\frac{\tan\theta}{\theta} \approx \frac{\sin\theta}{\theta} = 1$

(ii) $\therefore$ If $\theta = 10^\circ$, $\sin\theta = 0.1736$, $\tan\theta = 0.1763$; difference is 0.0027

$\cos\theta = \cos 10^\circ = 0.9848$

Thus there would be an error of 1.51 % when $y \approx d$. If distance x is measured along the horizontal instead of the sloping bed, then an error of order of 2% occurs. If $\theta = 11^\circ$ *or* $S_o = 0.20$ which is an extremely steep slope in open channels. However, there is exception in cases such as spillways, falls, and chutes.

Spillways will have slopes of $\theta = 45^\circ$ to 60°.

In the uniform flow, $S_f = S_w = S_o$. According to the law of conservation of energy, the total energy head at the upstream section should be equal to the total energy head at the downstream section plus the loss of energy h_f between the two sections. In other words

$$d_1 \cos\theta + \alpha_1 \frac{\overline{V}_1^2}{2g} + z_1 = d_2 \cos\theta + \alpha_2 \frac{\overline{V}_2^2}{2g} + z_2 + h_{f1\text{-}2}$$

This equation applies to parallel or gradually varied flow. For a channel of small slope, it may be simplified as $z_1 + y_1 + \alpha_1 \frac{\overline{V}_1^2}{2g} = z_2 + y_2 + \alpha_2 \frac{\overline{V}_2^2}{2g} + h_{f1\text{-}2}$

The above equations are known as the energy equation. If $\alpha_1 = \alpha_2 = 1$ and, $h_f = 0$ then the above equation reduces to $y_1 + \frac{\overline{V}_1^2}{2g} + z_1 = y_2 + \frac{\overline{V}_2^2}{2g} + z_2 = \text{constant}$

This is the well known Bernoulli (energy) equation.

Problem

(This may be attempted after learning about Hydraulic Jumps).

The reservoir level upstream of 50 m wide spillway for a flow of 1350 m^3/s is at elevation 250 m. The downstream river level for this flow is at El. 120. Determine the invert level of the stilling basin having the same width as the spillway so that the hydraulic jump is formed in the stilling basin. Assume that the losses in the spillway are negligible and also find downstream depth, Froude number, y_1, y_2, F_1, F_2 and ΔE and power dissipated in this system.