3.3 Basic Momentum Equation

The momentum of the flow passing a channel section per unit time is expressed by $\frac{\beta\gamma Q\overline{V}}{g}$, in which β is the momentum coefficient,

$\gamma\left(=\rho g=1000\frac{kg}{m^3}*9.806\frac{m}{s^2}=9.806\text{ kN}\right)$ is the specific weight of water, Q is the discharge in m^3s^{-1}, and $\overline{V}$ is the mean velocity in m^3s^{-1}.

As per Newton's second law of motion, the rate of change of momentum in the body of water in a flowing channel is equal to the resultant of all the external forces that are acting on the body. Applying this to a channel of large slope (Figure), the following expression for the rate of change of momentum in the body of water confined between sections 1 and 2 can be written as

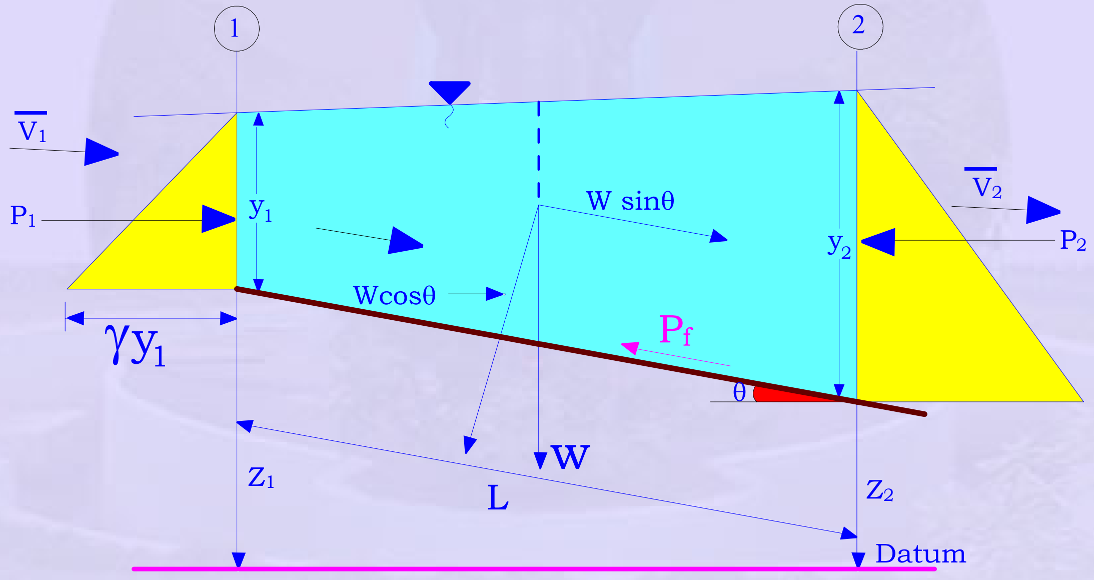

Figure - Momentum equation

$$\frac{\gamma Q}{g}\left(\beta_2\overline{V}_2-\beta_1\overline{V}_1\right)=P_1-P_2+W\sin\theta-P_f$$

in which subscripts refer to sections 1 and 2; P_1 and P_2 are the resultants of pressure forces acting on the two sections; W is the weight of water bounded between the sections; and P_f is the total external force due to friction and resistance acting along the surface of contact between the water and the channel. The above equation is known as the momentum equation and was first suggested by Belangar.

For gradually varied flow, the values of P_1 and P_2 in the momentum equation may be computed by assuming a hydrostatic pressure distribution. For a curvilinear or rapidly varied flow, however, the pressure distribution is no longer hydrostatic; hence the values of P_1 and P_2 cannot be so computed but are to be corrected. For simplicity, P_1 and P_2 may be replaced, respectively, by $\beta_1' P_1$ *and* $\beta_2' P_2$ *in which* β_1' *and* β_2' are correction coefficients at the two sections. These coefficients are called pressure distribution coefficients. Since P_1 and P_2 are forces, the coefficients may be specifically called force coefficients. It can be shown that the force coefficient may be expressed as

$$\beta' = \frac{1}{A\overline{z}} \int_0^A h \, dA = 1 + \frac{1}{A\overline{z}} \int_0^A c \, dA$$

in which $\overline{z}$ is the depth of the centroid of the water area A below the free surface, h is the pressure head on the elementary area dA, and c is the pressure - head correction factor. It can be shown that it is > 1.0 for concave flow, < 1.0 for convex flow, and equal to 1.0 for parallel flow.