6.3 Comparison Between Momentum and Energy Equation

Theoretically when the flow is gradually varied, energy and momentum equation should yield same results. Consider a gradually varied flow. The pressure distribution in the sections is taken as hydrostatic, the channel bed slope as small. For a rectangular channel of small slope and width b, in a short reach the expression for pressure forces can be written as

$$P_1 = \frac{1}{2}\gamma b y_1^2$$

and $$P_2 = \frac{1}{2}\gamma b y_2^2$$

If Force due to friction can be written as $P_f = \gamma h'_f b\bar{y}$

in which h'_f is the friction head and $\bar{y}$ is the average depth, or ($y_1 + y_2$) / 2. The discharge through the reach is equal to

$$Q = \frac{1}{2}\left(\bar{V}_1 + \bar{V}_2\right) b\bar{y}$$

Also, the weight of the body of water is

$$W = \gamma b\bar{y}L$$

and $$\sin\theta = \frac{z_1 - z_2}{L}$$

Then the momentum equation, after substituting these expressions simplifies (see box) as

$$z_1 + y_1 + \beta_1 \frac{\bar{V}_1^2}{2g} = z_2 + y_2 + \beta_2 \frac{\bar{V}_2^2}{2g} + \bar{h}'_f$$

$$z_1 + y_1 + \alpha_1 \frac{\bar{V}_1^2}{2g} = z_2 + y_2 + \alpha_2 \frac{\bar{V}_2^2}{2g} + h_{f_{1\text{-}2}}$$

This equation appears to be practically the same as the energy equation (Bernoulli equation). However, the energy loss given by momentum equation is due to external forces whereas the loss given by energy equation is due to internal forces. One is a vector quantity and other is scalar quantity. However, if the flow is uniform, then $h_f = h'_f$ if the difference between α and ρ is ignored. Similarity ends here. There are cases where either momentum equation or energy equation can be used with the continuity equation.

Momentum Application

$$Q = \left(\frac{\overline{V}_1+\overline{V}_2}{2}\right) b \left(\frac{y_1+y_2}{2}\right) = b\overline{y}\left(\frac{\overline{V}_1+\overline{V}_2}{2}\right)$$

$W = \gamma b \overline{y} L$ = Specific weight * (Volume)

$$\sin\theta = \frac{z_1 - z_2}{L}$$

$$b\overline{y}\frac{\gamma}{g}\left(\frac{\overline{V}_1+\overline{V}_2}{2}\right)\left[\beta_2 \overline{V_2} - \beta_1 \overline{V_1}\right] = \frac{1}{2}\gamma b y_1^2 - \frac{1}{2}\gamma b y_2^2 + \gamma b \overline{y}\, L\sin\theta - \gamma h_f' b\overline{y}$$

$$\frac{\gamma b\overline{y}}{2g}\left[\beta_2 \overline{V_1}\,\overline{V_2} + \beta_2 \overline{V_2}^2 - \beta_1 \overline{V_1}^2 - \beta_1 \overline{V_1}\,\overline{V_2}\right] = \frac{1}{2}\gamma b y_1^2 - \frac{1}{2}\gamma b y_2^2 + \gamma b\overline{y}\, L\frac{z_1 - z_2}{L} - \gamma h_f' b\overline{y}$$

divided by γb

$$\frac{\overline{y}}{2g}\left[\beta_2 \overline{V_2}^2 - \beta_1 \overline{V_1}^2 + \beta_2 \overline{V_1}\,\overline{V_2} - \beta_1 \overline{V_1}\,\overline{V_2}\right] = \frac{1}{2}y_1^2 - \frac{1}{2}y_2^2 + \frac{y_1+y_2}{2}z_1 - \frac{y_1+y_2}{2}z_2 - h_f'\left(\frac{y_1+y_2}{2}\right)$$

$$\frac{y_1+y_2}{2}\left[\beta_2 \frac{\overline{V_2}^2}{2g} - \beta_1 \frac{\overline{V_1}^2}{2g} + \beta_2 \frac{\overline{V_1}\,\overline{V_2}}{2g} - \beta_1 \frac{\overline{V_1}\,\overline{V_2}}{2g}\right] = \frac{1}{2}y_1^2 - \frac{1}{2}y_2^2 + \frac{y_1}{2}z_1 + \frac{y_2}{2}z_1 - \frac{y_1}{2}z_2 - \frac{y_2}{2}z_2 + \overline{h}_f'\left(\frac{y_1+y_2}{2}\right)$$

Simplifying

$$\beta_2 \frac{\overline{V_2}^2}{2g} - \beta_1 \frac{V_1^2}{2g} + \frac{\beta_2 \overline{V_1}\,\overline{V_2} - \beta_1 \overline{V_1}\,\overline{V_2}}{2g} = (y_1 - y_2) - z_1 - z_2 - \overline{h}_f'$$

If $\beta_1 \approx \beta_2$ we can neglect $\dfrac{\beta_2 \overline{V_1}\,\overline{V_2} - \beta_1 \overline{V_1}\,\overline{V_2}}{2g} \approx 0$

$$z_1 + y_1 + \beta_1 \frac{\overline{V_1}^2}{2g} = z_2 + y_2 + \beta_2 \frac{\overline{V_2}^2}{2g} + \overline{h}_f'$$