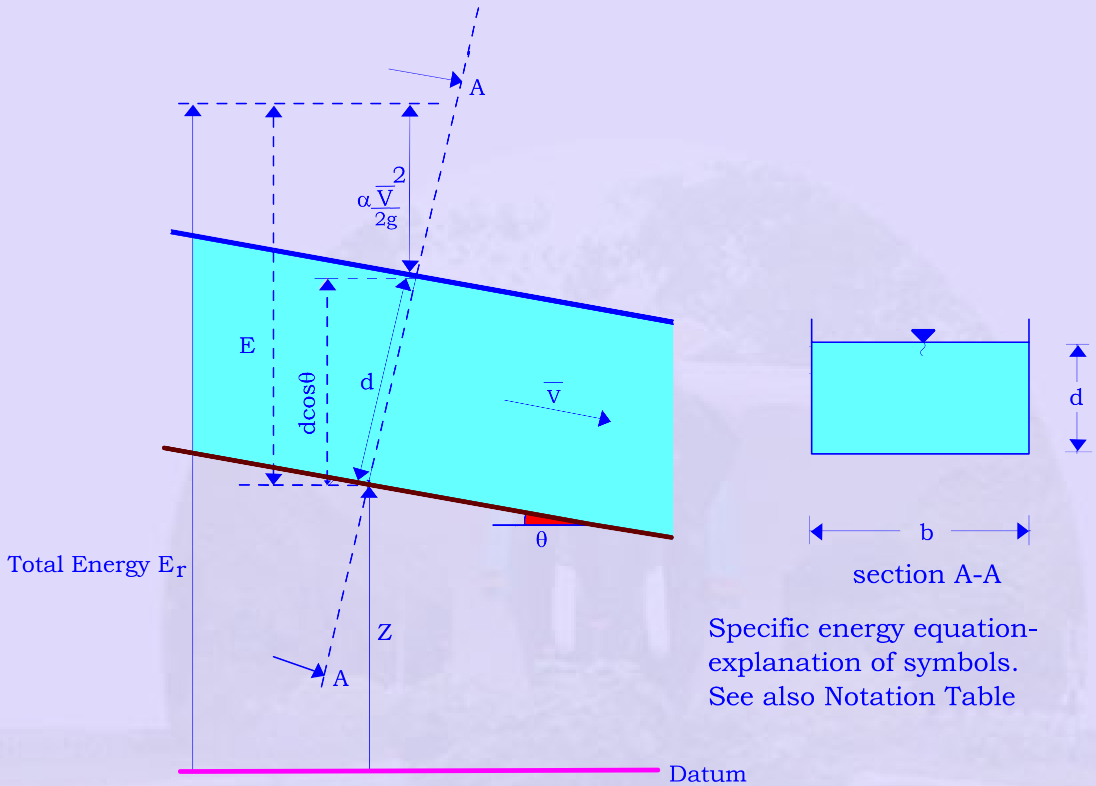
8.1 Specific energy equations for rectangular channels

Specific energy in Open channel is defined as the energy per $\frac{\text{N m}}{\text{N}}$ of water at any section of a channel measured with respect to the channel bottom. Thus it is the total energy with z being zero.



Notations
E_r = Total energy above datum = $z + d\cos\theta + \alpha\left(\frac{\overline{V}^2}{2g}\right)$
E = Specific energy = $d\cos\theta + \alpha\left(\frac{\overline{V}^2}{2g}\right) = d\cos\theta + \alpha\left(\frac{q^2}{2gd^2}\right)$
Q = Discharge, b = channel width, d = flow depth,
q = Discharge per unit width = Q/b,
tan θ = Bed slope, α = Velocity coefficient, g = Acceleration due to gravity

Thus specific energy can be written as

$$E = d\cos\theta + \alpha\frac{\overline{V}^2}{2g}$$

The concept of specific energy as it applies to open channels with small slopes is given below.

Total energy equation is

$$\frac{p}{\gamma}+\frac{\bar{v}^2}{2g}+z=\text{constant}$$

In other words it can be rewritten as

$$y+\frac{\bar{v}^2}{2g}+z=\text{constant}$$

If $z=0$ then

$$E=y+\frac{\bar{v}^2}{2g}$$

which indicates that the specific energy is the sum of the depth of water and the velocity head.

8.1.1 Specific energy diagram

Solution of the specific energy equation for rectangular channels

Consider a specific energy equation for the case of a rectangular channel.

$$E=y+\frac{\bar{v}^2}{2g}$$

Discharge $Q=\bar{V}A$

Therefore $\bar{V}=\frac{Q}{A}$

$$\bar{V}^2=\left(\frac{Q}{A}\right)^2=\frac{Q^2}{b^2y^2}$$

in which b is the width of the channel and y is the depth of flow.

Substituting this in the specific energy equation it can be written as

$$(E-y)=\frac{Q^2}{2gy^2b^2}$$

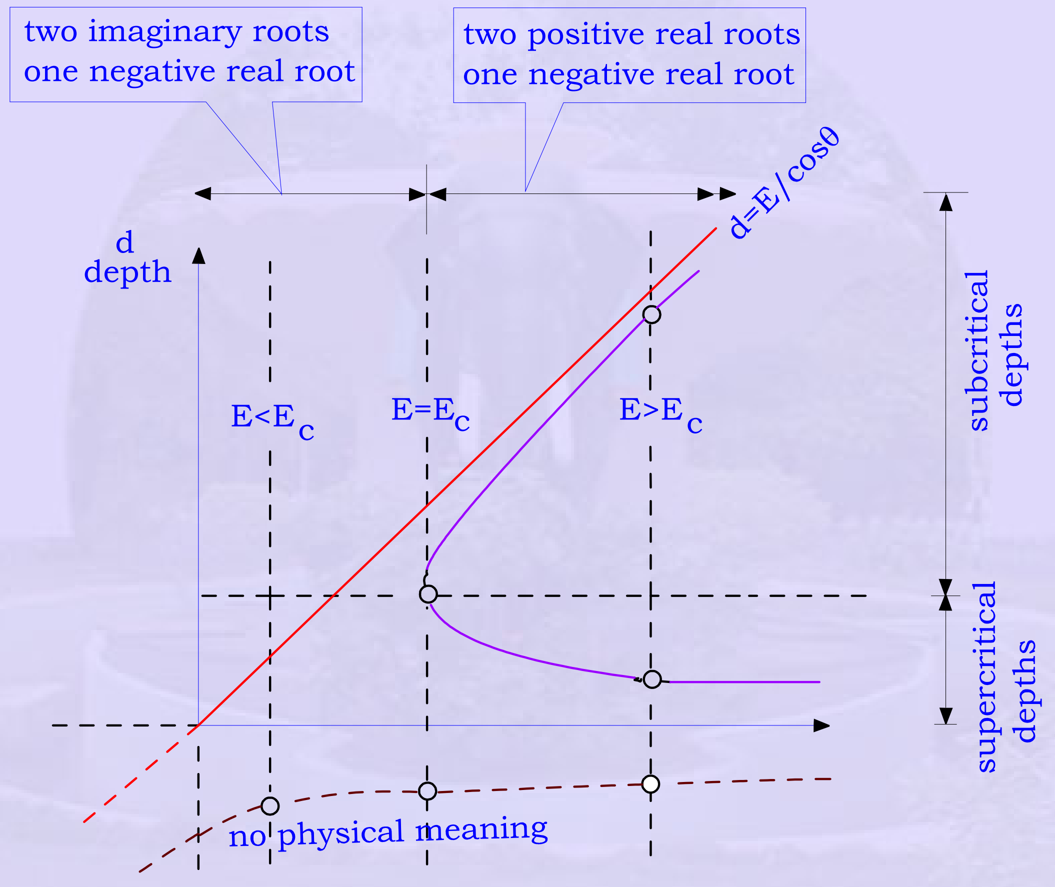
Defining $q=\frac{Q}{b}$

Then $(E-y)y^2=\frac{Q^2}{2gb^2}=\frac{q^2}{2g}$ a constant

$$(E-y)y^2=\text{constant}$$

The above algebraic equation is a cubic equation and there are three routes for y for given specific energy E. Out of which two are positive roots and one is negative root. i.e. y<0 which is physically impossible. Therefore it is only an imaginary solution. The two positive depths are called alternative depths. Normally indicated as y_1 and y_2 for supercritical and sub critical condtions and are known as low stage and high stage values of depths.

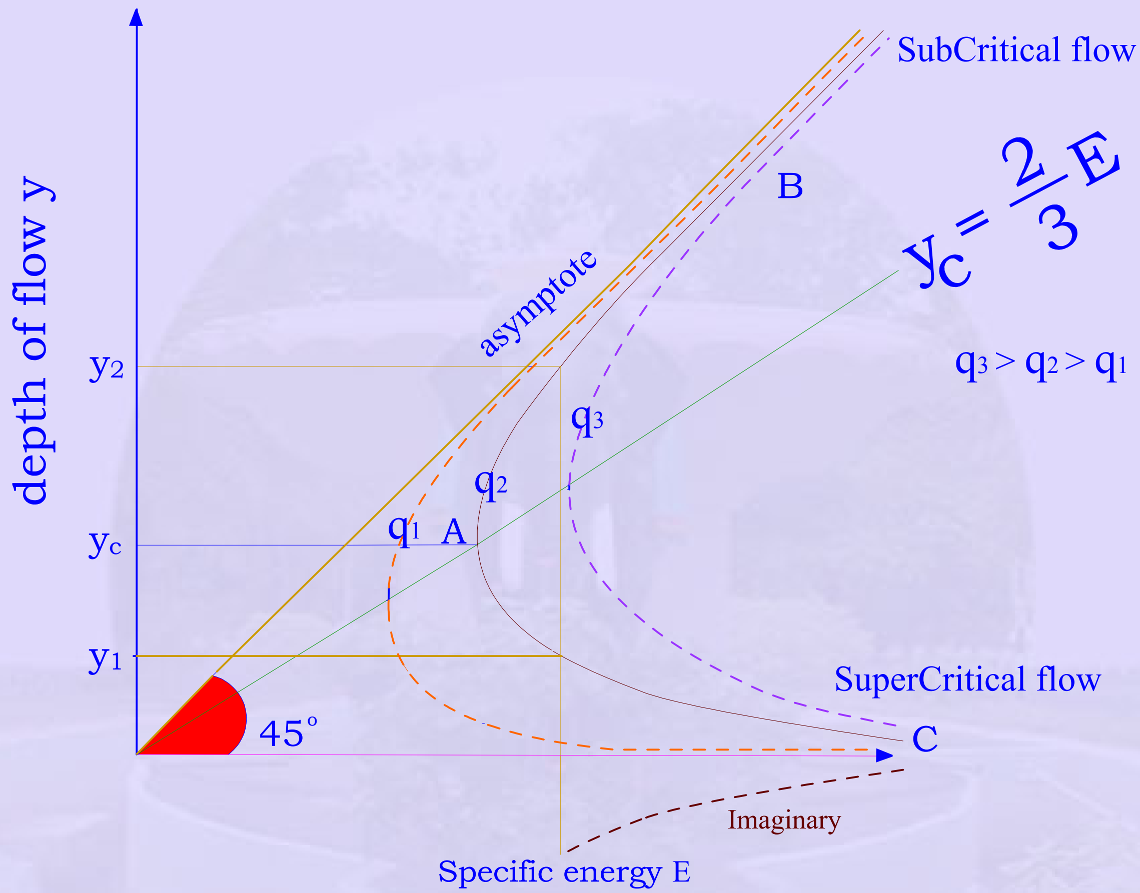
This is graphically shown in Figure, where the specific energy is plotted against the depth, for a given discharge per unit width, rendering the familiar representation.



Specific energy where $E = d\cos\theta + \alpha\,(q^2/2gd^2)$

When $E > E_c$ (minimum energy for a given q) three real unequal roots are obtained: two positive ones (sub critical and supercritical depths) and a third one negative (no physical meaning). When $E = E_c$ the two positive roots become equal and this depth is the critical depth. When $E < E_c$ the two positive roots become imaginary and the third one remains negative.

Figure below shows the variation of the specific energy as a function of depth when the discharge per unit width changes. when q increases the corresponding critical depths increase and the positive and negative limbs of the function move away from the origin. The opposite applies when q decreases. When q=0 the critical depth is equal to zero, the sub critical depth equals $E / cos\,\theta$ and the supercritical depth (and the negative root) are equal to zero.



Specific energy diagram

The Specific energy curve is confined between two asymptotes namely y = E and y = 0. The first asymptote is at 45° with respect to abscissa. However, if the effect of the bed slope of the channel is considered the angel will be different from 45°.

For a given Q, specific energy curve has two limbs BA and AC.

Line BA represents Sub critical flow

Line AC represents Super critical flow

C represents Critical flow.

For a given Specific energy E there are three possible depths: Two positive values and one negative value. Two positive values are y_1 and y_2 respectively representing Super critical and Sub critical depths. The minimum value of specific energy for the given discharge represents the critical flow condition.

The locus of this represented by $y_c = \frac{2}{3}E$

For different values of discharges namely Q_1, Q_2, Q_3 different specific energy curves would be there.

The minimum specific energy represents the critical condition.

$$E = d\cos\theta + \frac{\alpha\overline{V}^2}{2g}$$

$$\frac{dE}{dd} = \cos\theta + \frac{\alpha}{2g}2\overline{V}\frac{d\overline{V}}{dd}$$

$$= \cos\theta - \frac{2\alpha Q^2}{2gA^3}\frac{dA}{dd} = 0$$

$$\cos\theta = \frac{2\alpha Q^2}{2gA^3}T$$

$$D\cos\theta = \frac{\alpha Q^2}{gA^2} = \frac{\alpha\overline{V}^2}{2g}$$

$$1 = \frac{\alpha\overline{V}^2}{gD\cos\theta}$$

Making *Froude Number, F = 1 for critical conditions, F can be defined as*

$$F = \frac{\overline{V}}{\sqrt{gD\frac{\cos\theta}{\alpha}}}$$

This is for non rectangular channel.

If $\alpha = 1$, *and* θ is very small then it can be written as

$$F = \frac{\overline{V}}{\sqrt{gD}}$$

In which D is the hydraulic mean depth.

Thus the specific energy is minimum when the flow is critical.

$$\left(\frac{\overline{V}}{\sqrt{gD}}\right)^2 = 1$$

$$\therefore \frac{\overline{V}^2}{g} = D \quad or \quad \frac{\overline{V}^2}{2g} = \frac{D}{2}$$