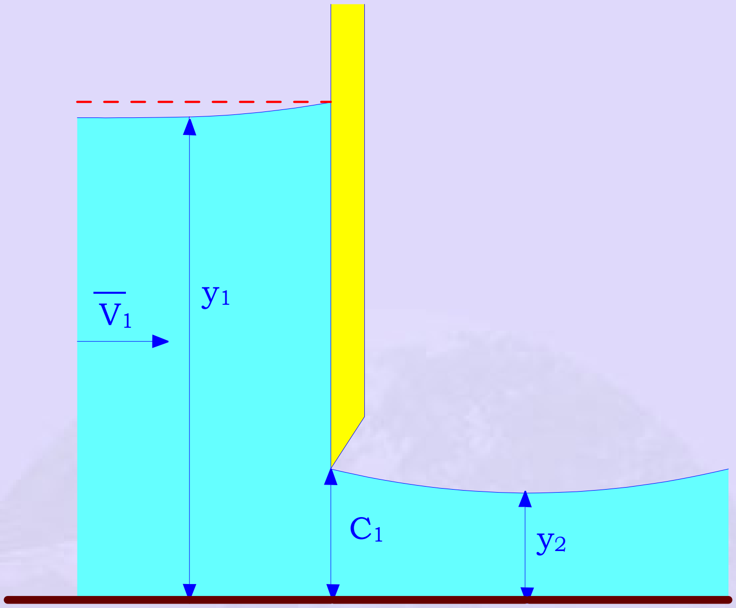
8.2 Application of Specific energy

Transition Problem:



Sluice gate

Example 1:

Flow below a sluice gate

Problem: If $y_1 = 2.5$ m, $y_2 = 0.60$ m, $b = 3.5$ m, determine the discharge Q.

Solution:

Apply Bernoulli equation beteween sections 1 and 2, assuming losses are negligible.

$$y_1 + \frac{\overline{V}_1^2}{2g} + z_1 = y_2 + \frac{\overline{V}_2^2}{2g} + z_2$$

$z_1 = z_2$ and width b is constant

$$Q = A_1\overline{V}_1 = A_2\overline{V}_2$$
$$= b\,y_1\overline{V}_1 = b\,y_2\overline{V}_2$$
$$= 3.5\,(2.5)\overline{V}_1 = 3.5(0.6)\overline{V}_2$$
$$8.75\,\overline{V}_1 = 2.10\,\overline{V}_2$$
$$\overline{V}_2 = \left(\frac{8.75}{2.1}\right)\overline{V}_1$$
$$\overline{V}_2 = 4.16\overline{V}_1$$
$$\frac{\overline{V}_2^2}{2g} = (4.16)^2\frac{\overline{V}_1^2}{2g} = 17.36\frac{\overline{V}_1^2}{2g}$$

Substituting the values into specific energy equation

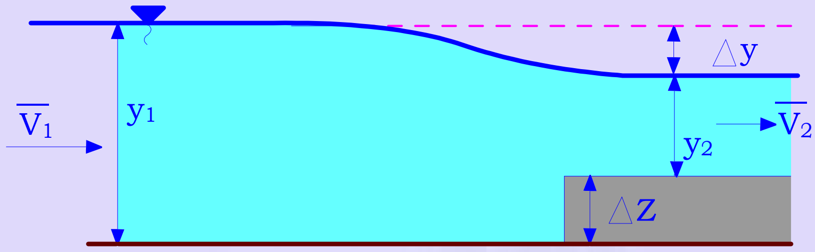
$$2.5 + \frac{\overline{V}_1^2}{2g} = 0.6 + (4.16)^2\frac{\overline{V}_1^2}{2g}$$
$$2.5 + \frac{\overline{V}_1^2}{2g} = 0.6 + 17.36\frac{\overline{V}_1^2}{2g},$$
$$16.36\frac{\overline{V}_1^2}{2g} = 2.5 - 0.6$$
$$\overline{V}_1 = \sqrt{\frac{(2.5-0.6)*2*9.81}{16.36}} \qquad \overline{V}_1 = 1.5095\,\text{ms}^{-1}$$
$$Q = 13.2081\,\text{m}^3/\text{s}$$
$$\overline{V}_2 = 6.2795\,\text{m/s}$$

Froude number in the downstream $F_2 = \dfrac{\overline{V}_2}{\sqrt{g y_2}}$

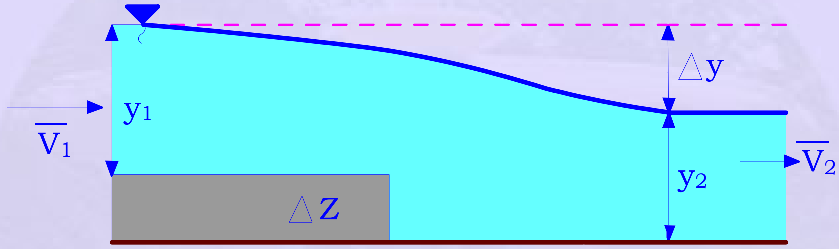
$$= \frac{6.28}{\sqrt{9.81*0.6}} = 2.59$$

Example 2:

Consider a transition with a vertical step of height Δz in bed, in a rectangular channel of constant width b. upward step Δz is considered as positive. What is the depth over the step?



Positive step of △z height



Negative step of △z height

$$y_1 + \frac{\overline{V}_1^2}{2g} = y_2 + \frac{\overline{V}_2^2}{2g} + \Delta z$$

$$\frac{Q}{b} = q, \quad y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2} + \Delta z$$

$$y_2 + \frac{q^2}{2gy_2^2} = y_1 + \frac{q^2}{2gy_1^2} - \Delta z$$

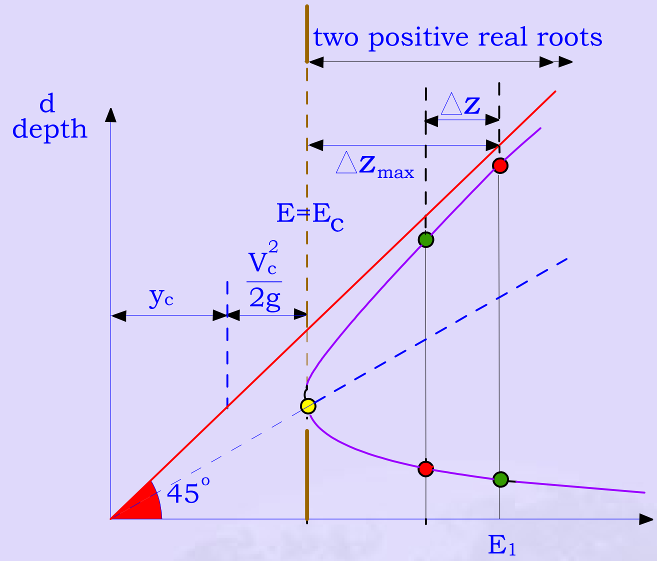
$$E_2 = E_1 - \Delta z$$

E_1 and Δz are known. E_2 is to be solved for y_2 by trial and error or using solution of cubic equation.

Note:

Subcritical flow can change over to supercritical or subcritical flow depending on the downstream conditions.

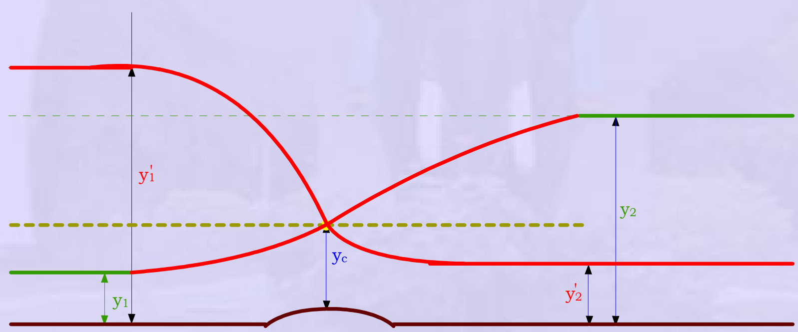
Similarly, supercritical flow can change over to supercritical or subcritical flow depending on the downstream conditions.



For a given specific energy E_1, the step height $\triangle Z = E_1 - E_2$

The maximum step height $\triangle Z_{max} = E_1 - E_c$

Note, if the step height is more than the $\triangle Z_{max}$ for the given q the choking occurs. In other words the given discharge cannot flow over the step until the specific energy increases.



Flow over a transition

Given specific energy E_1 has two depths namely y_1 and y_1' (initial and alternate depth).

The flow correspondingly it would be super critical and sub critical flows and could be vice versa.

In the downstream for the given specific energy E_2 two possible depths are y_2' and y_2 corresponding to super critical flow and sub critical flow respectively (alternate depths of E_2).

If the critical depth occurs on the step then there are four possible situations of water surface profiles.

Super critical to Sub critical	1. Super critical flow (y_1) changing over to y_2 subcritical through y_c causing classical hydraulical jump.
Super critical to Super critical	2. Super critical flow (y_1) changing over to y_2' through y_c. Thus it would be super critical to super critical.
Sub critical to Super Critical	3. Sub critical approach flow y_1' changing over to y_2' via critical depth y_c. Thus a hydraulic drop occurs.
Sub critical to Sub critical	4. The sub critical approach flow y_1' changing over to y_2 via critical depth y_c. The occurrance of one of the above type of profiles depends entirely on the downstream condition.

For a given discharge

$$E_1 - E_2 = \Delta E$$

$$E_1 = \Delta E + E_2$$

If the flow is critical on the step then $E_c = \frac{2}{3} E_1$

$$\Delta Z = E_1 - E_2$$

If E_2 is to be equal to critical flow, then $E_c = \frac{2}{3} E_1$

$$\Delta Z = E_1 - \frac{2}{3} E_1 = \frac{1}{3} E_1.$$

The step height is maximum.

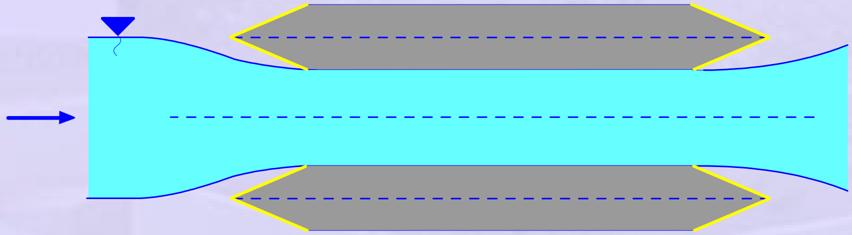
Depths y_1' and y_2' are known as alternate depths, and y_2 becomes y_c.

The water surface profile downstream of this depends on downstream controls.

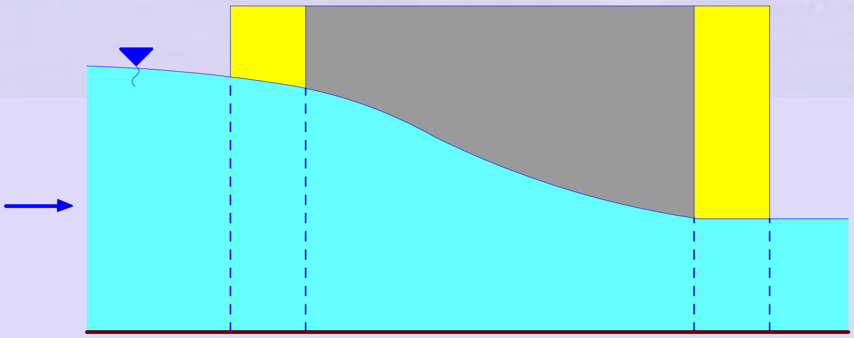
If applied to a case of constriction of width, then critical width of contraction.

$$y_c^3 = \frac{\left(q^2 * b\right)}{g} \text{ or } b_{min} = \left[\frac{g y_c^3}{q^2}\right]$$

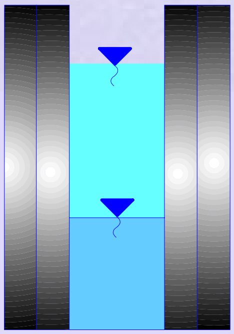
Minimum specific energy line on vertical is C which is the critical depth. Therefore Maximum constriction is obtained.



Flow positive pier - width reduces



Longitudinal water surface profile



Looking from downstream

Water surface drops due to constriction in width. Example near the bridge piers.

Exercise problems:

1. Show that when, in a rectangular channel with a horizontal bed F = 1 and $\frac{db}{dx} = 0$, the width must be a minimum and not a maximum. (Hint: Consider the variation of $\bar{v}$ and with b for F > 1 and F < 1).

8.2.1 Normalisation of the specific energy curves

Consider the specific energy equation

$$E = y + \frac{q^2}{2gy^2}$$

Dividing by the critical depth y_c, it can be written as

$$\frac{E}{y_c} = \frac{y}{y_c} + \frac{q^2}{2gy^2 y_c} \quad \text{but}$$

$$\left(\frac{q^2}{gy_c^3} = 1 \right)$$

$$\frac{q^2}{2gy^2 y_c} = \frac{y_c^2}{2y^2}$$

If $\frac{y}{y_c} = y'$ and $\frac{E}{y_c} = E'$

$$\therefore \quad E' = y' + \frac{y_c^2}{2y^2}$$

$$E' = y' + \frac{1}{2y'^2}$$

which is similar to general form of E' Vs y'.

when critical *depth* y_c is known from equation this specific energy curve can be used for obtaining length scale for modelling.

It can be shown for rectangular channel that

$$F^2 = \frac{\bar{v}^2}{gy} = \frac{q^2}{gy^3} = \frac{gy_c^3}{gy^3} = \left(\frac{y_c}{y} \right)^3$$

Problem:

In case of Simple upward step (Δz being +ve),

For a geometrically similar model $\frac{y_1}{y_c}, \frac{y_2}{y_c}, \frac{\Delta z}{y_c}$ are same in model and prototype each case. Dynamic similarity condition should exist while the Froude similitude, and if $\frac{y}{y_c}$ are equal for two situations, then the ration of discharge is equal to $q_r = y_r^{3/2}$.

Determine an expression for slope of the straight line to which the upper limb of the specific energy curve is an asymptote for a channel having a bottom slope of θ.

Solution:

Let us consider Bernoulli's equation.

$$z + y + \frac{\overline{V}^2}{2g} = H$$

But $\overline{V} = \frac{Q}{A}$

Let d $\rightarrow$ *depth of flow normal to the channel bottom and* $\theta \rightarrow$ slope *of the channel.*
then y = dcos θ. Therefore the specific energy

$$\therefore E = d \cos\theta + \frac{\alpha Q^2}{2gA^2}$$

Since $d = y \cos\theta$

$$\therefore E = y \cos^2\theta + \frac{\alpha Q^2}{2gA^2}$$

Consider uniform

$$E - y\cos^2\theta = \frac{\alpha Q^2}{2gA^2} \quad \text{constant (approximately)}$$

Assume *angle between slope of straight line and horizontal axis as* ϕ

$E - y\cos^2\theta = 0$ *is one asymptote*

$$tan\ \phi = \frac{y}{E}$$

$$E = y\ cos^2\theta$$

$$1 = \frac{y}{E} cos^2\theta \quad \textit{from figure}$$

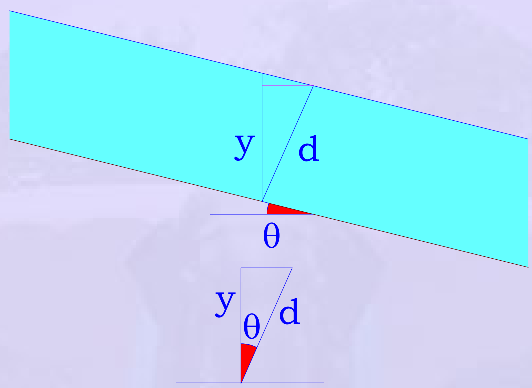
$$1 = tan\ \phi\ cos^2\theta$$

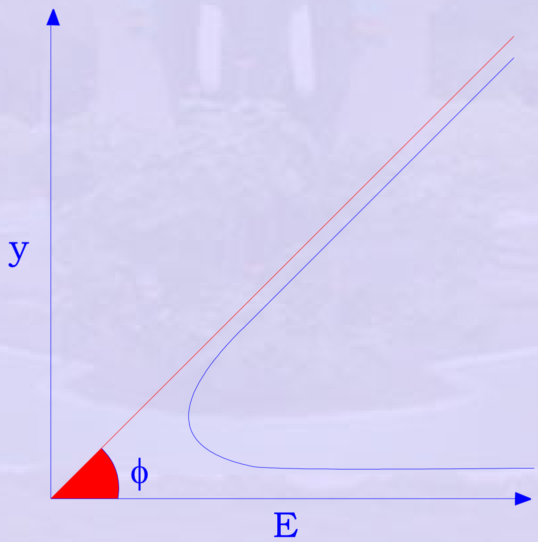
$$tan\ \phi = \frac{1}{cos^2\theta}$$

The expression for the slope of straight line to which upper limb of specific energy curve is

$$\phi\ = tan^{-1}\left(\frac{1}{cos^2\theta}\right)$$

The *angle* (ϕ) *depends upon the bed slope of the channel.*





Problem:

Plot the specific energy vs depth curves for Q = 400, 600 and 800 m^3/s in a trapezoidal channel having bottom width of 20 m and the side slopes of 2(H) : 1(V). Assume the bottom slope as small. From these curves, determine the critical depth for each discharge. Write a computer program to obtain the above.