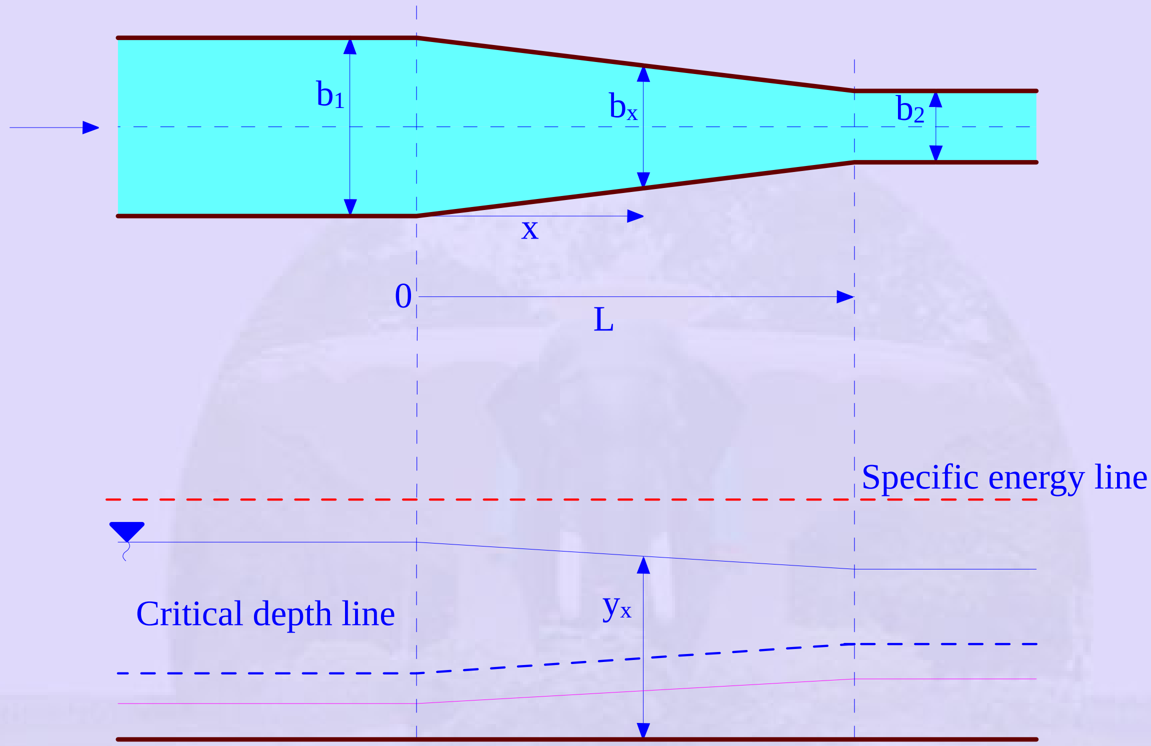
9.2 Transition: A Field Example

1. A horizontal channel converges from width b1 to b2 over a distance L. Approach flow is sub critical. No hydraulic drop is permitted. Given Q, y_1. Determine the water surface profile.

Solution



Steps

Given Q and depth y_1

1. $Q = A_1\overline{V}_1, \quad \therefore \overline{V}_1 = \dfrac{Q}{b_1 y_1}$

$$\therefore E_1 = y_1 + \frac{\overline{V}_1^2}{2g},$$

2. It is assume that no energy loss takes place along the transition

3. $E_2 = y_2 + \dfrac{\overline{V}_2^2}{2g} = y_2 + \dfrac{Q_1^2}{2gy_2^2 b_2^2}$

4. $E_1 = E_2, \quad \therefore$ obtain subcritical depth y_2 by trial and error or by direct solution

5. Let Sub critical depth at any section x be is y_x

$$E_1 = y_x + \frac{Q^2}{2gy_x^2 b_x^2}$$

$$b_x = b_1 - \frac{(b_1 - b_2)x}{L}$$

6. Solve for y_x for various x.

   a. Plot the profile:

   In this case as the transition is a straight wall transition, water surface can be joined between y_1 and y_2.

2. In the above problem if a hydraulic drop is permitted at a distance x, determine the water surface profile what would be the constriction width?

Solution

Hydraulic drop means flow changes from sub critical to super critical via y_c

Step1:

$$y_c = \sqrt[3]{\frac{Q^2}{gb_x^2}}$$

Step2:

$$E_1 = y_1 + \frac{Q^2}{2gb_1^2 y_1^2}$$

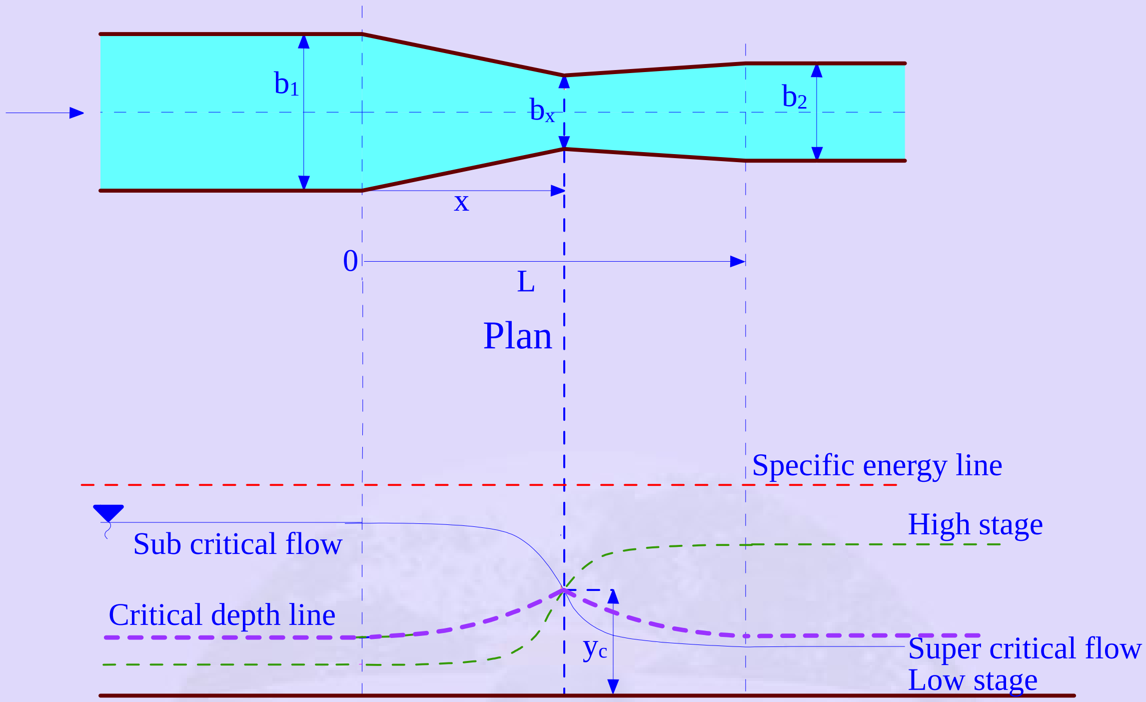
$$y_c = \frac{2}{3}E_1 \qquad \therefore b_x \text{ can be determined.}$$

Thus maximum constriction at x is known. After determining the b_x obtain super critical depths (low stage depths) in the downstream of this constriction. If b_{x1} is the width at a distance x_1 from the constriction then,

$$b_{x1} = b_x + \frac{(b_2 - b_x)}{L} x_1$$

$$E_2 = E_{x1} = y_{x1} + \frac{Q^2}{2gb_{x1}^2 y_{x1}^2}$$

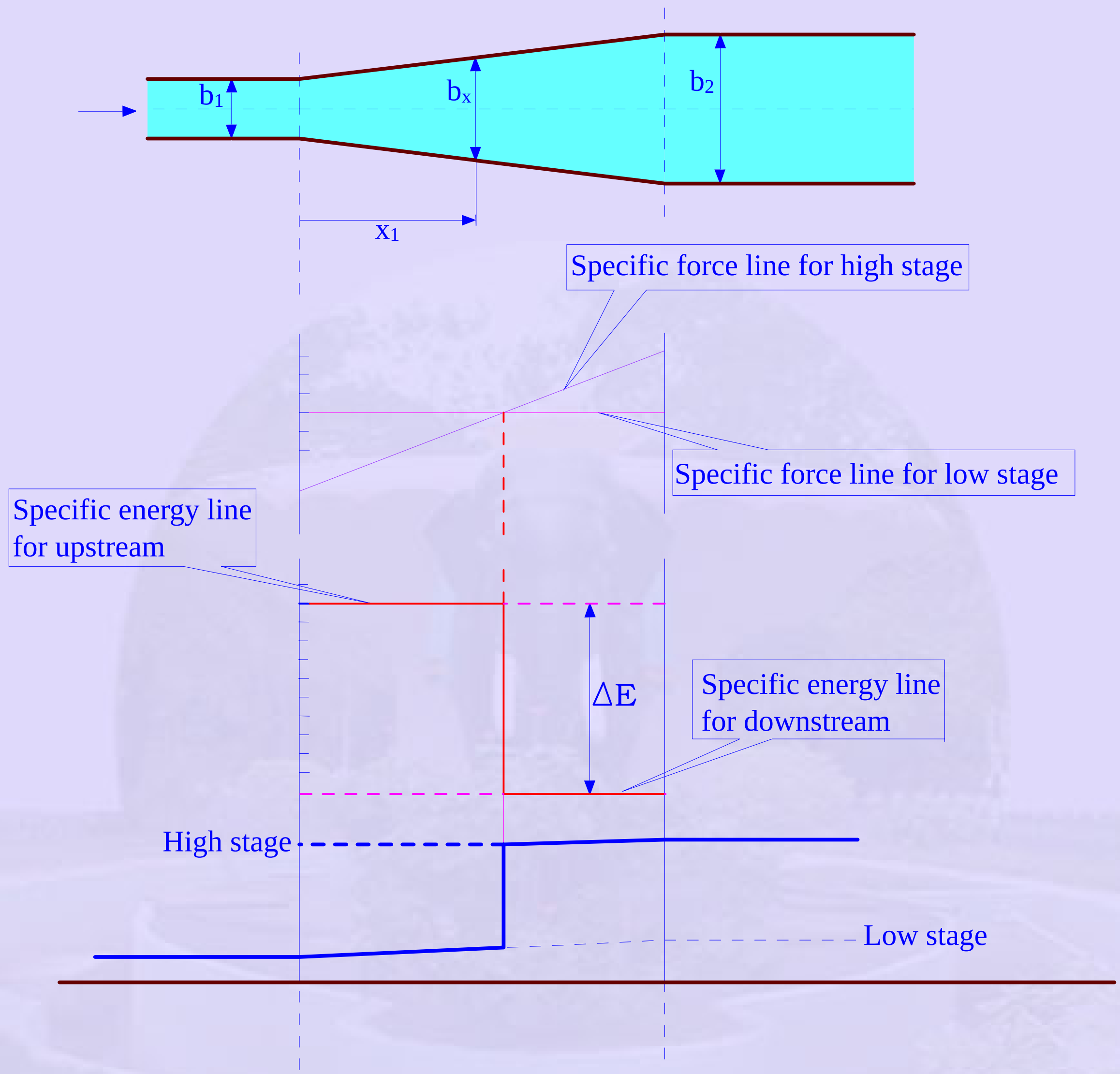
Solve for y_{x1} for super critical flow conditions. Figure shows the typical water surface profile.



Water surface profile with hydraulic drop

3. Super critical flow occurs in the approach channel of a transition in which b_1 changes to b_2 as shown in figure. Locate the hydraulic jump if it occurs.

Solution



a) When jump occurs energy loss takes place

$$E_1 - E_2 = \Delta E$$

$$E_1 = y_1 + \frac{\overline{V}_1^2}{2g} = y_1 + \frac{Q^2}{2g y_1^2 b_1^2}$$

b) y_1 is given Q is given $\therefore F_1 = \frac{\overline{V}_1}{\sqrt{g y_1}}$

c) Given Q, b_2, y_2 at the downstream section, y_2 should be sub critical depth if the jump has to occur.

d) Jump occurs but (i) can occur in the transition reach, (ii) not in the transition reach.

Step1: Compute E_1 and plot the line.
Compute E_2 and plot the line.

Step 2: Compute specific force $M_1 = \bar{z}A_1 + \frac{Q^2}{gA_1}$

Similarly compute $M_2 = \bar{z} + \frac{Q^2}{gA_2}$

Step 3:

When the specific force M_1=M_2, the hydraulic jump forms. It may be noted that jump will have certain length. In this calculation it is assumed that it occurs in a section.

x	bx	low stage depth for specific energy E1	Specific force for low stage	Specific force for high stage	Remarks
0	b_1	y_1	M_1	M_2	
2					
5					
-					
-					
x	b_x				

From the above computation plot a force lines and the intersection gives the location of the jump.

The location of the jump is at the section where the specific forces are equal. Therefore solving these two algebraic equations for specific forces simultaneously the location of the jump x can be determined.

4. Elimination of the jump by a hump

In the above problem modify the transition to eliminate the jump by providing a hump. Obtain the hump profile.

Solution

1. Assume a smooth water surface profile between approach flow depth and the downstream depth. Thus an elevation H_x of the water surface profile is known.
2. Assume that E_1 to E_2 loss is distributed linearly over transition.

3. It can be written that

$$H_x = y_x + z_x,$$

$$\therefore z_x = H_x - y_x$$

At any point x the specific energy is given by

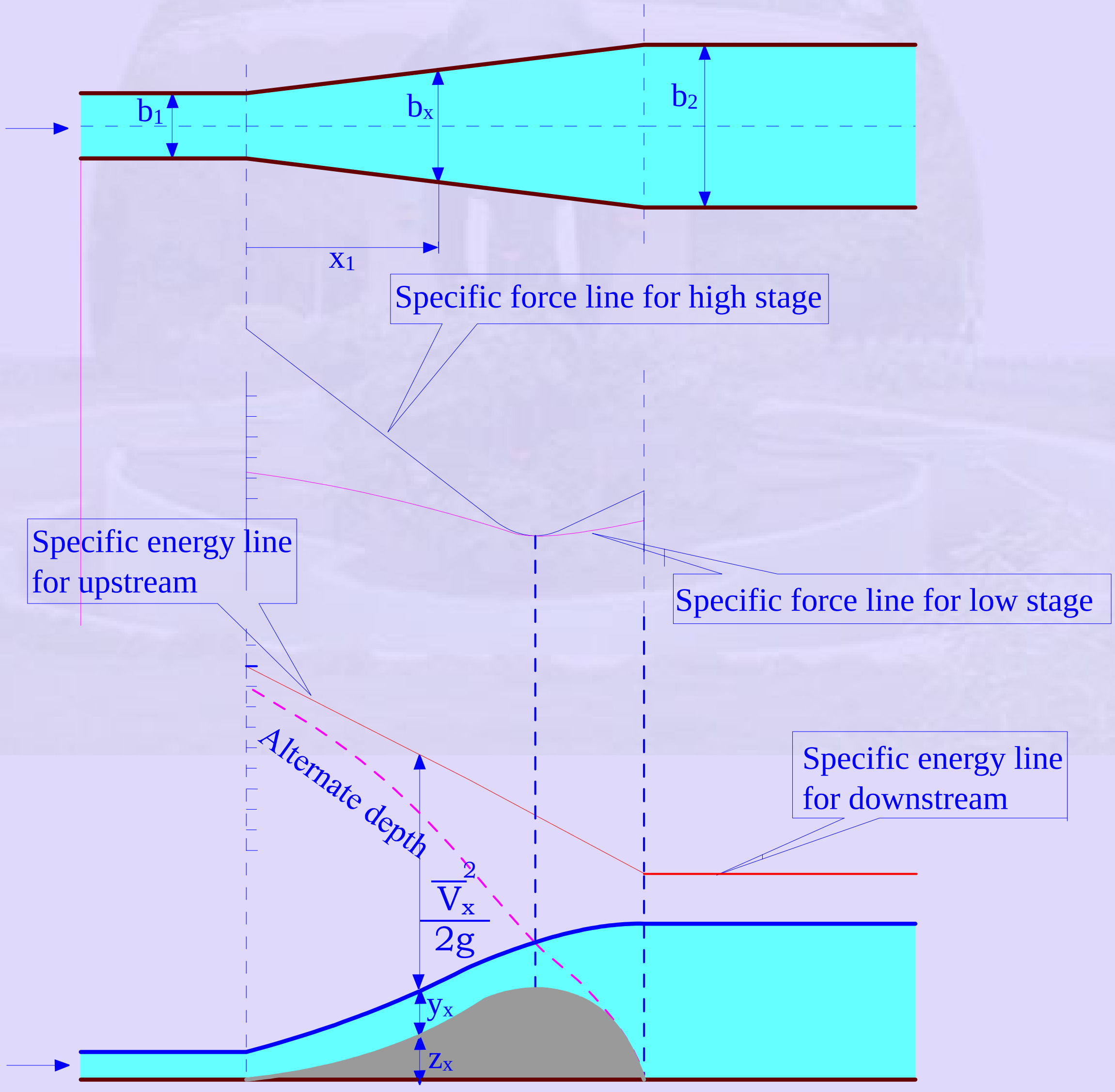
$$\left[E_1 - \frac{(E_1 - E_2)}{L}x\right] = \frac{\overline{V}_x^2}{2g} + H_x$$

In the above equation only unknown is velocity. Hence calculate the velocity.

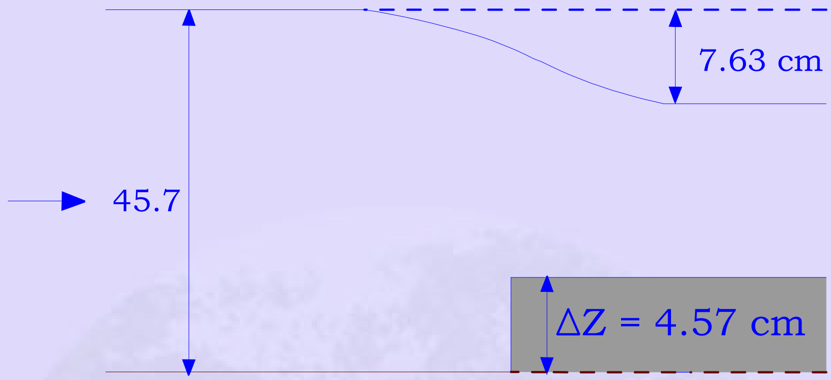
But $\overline{V}_x = \frac{Q^2}{A_x^2} = \frac{Q^2}{b_x^2\ y_x^2\ 2g}$. Calculate y_x knowing $b_x = b_1 + \frac{(b_2 - b_1)}{L}x$

$\therefore$ Obtain z_x from the known H_x *by subtracting the depth* y_x and plot the z as a function of x to obtain the hump profile.

Alternative solution for eliminating the jump is to increase the roughness In other words increasing the friction. Also by changing the width.



5. The flow is taking part a section shown in Figure. The step height is 4.57 cm. The upstream depth 45.7 cm. The water surface drops by 7.63 cm from its original level on the step. Determine the discharge.



Flow over a step

Solution

$$y_2 = 45.7 - 4.57 - 7.63 = 45.7 - 12.2 = 33.5\ cm$$

$$E_1 = y_1 + \frac{\overline{V}_1^2}{2g} \qquad E_2 = y_2 + \frac{\overline{V}_2^2}{2g}$$

$$by_1\overline{V}_1 = by_2\overline{V}_2 \qquad \therefore \frac{\overline{V}_1}{\overline{V}_2} = \frac{y_2}{y_1} = \frac{33.5}{45.7} = 0.733$$

$$\text{So } \overline{V}_1 = 0.733\overline{V}_2 \quad \text{or} \quad \overline{V}_2 = 1.364\ \overline{V}_1$$

$$\Delta z = \left(y_1 + \frac{\overline{V}_1^2}{2g} \right) - \left(y_2 + \frac{\overline{V}_2^2}{2g} \right)$$

$$4.57 = 45.7 - 33.5 + \frac{\overline{V}_1^2}{2g}\left[1 - \frac{\overline{V}_2^2}{\overline{V}_1^2} \right]$$

$$\frac{\overline{V}_1^2}{2g}\left[1 - \left(\frac{1}{0.733} \right)^2 \right] = 4.57 - 45.7 + 33.5$$

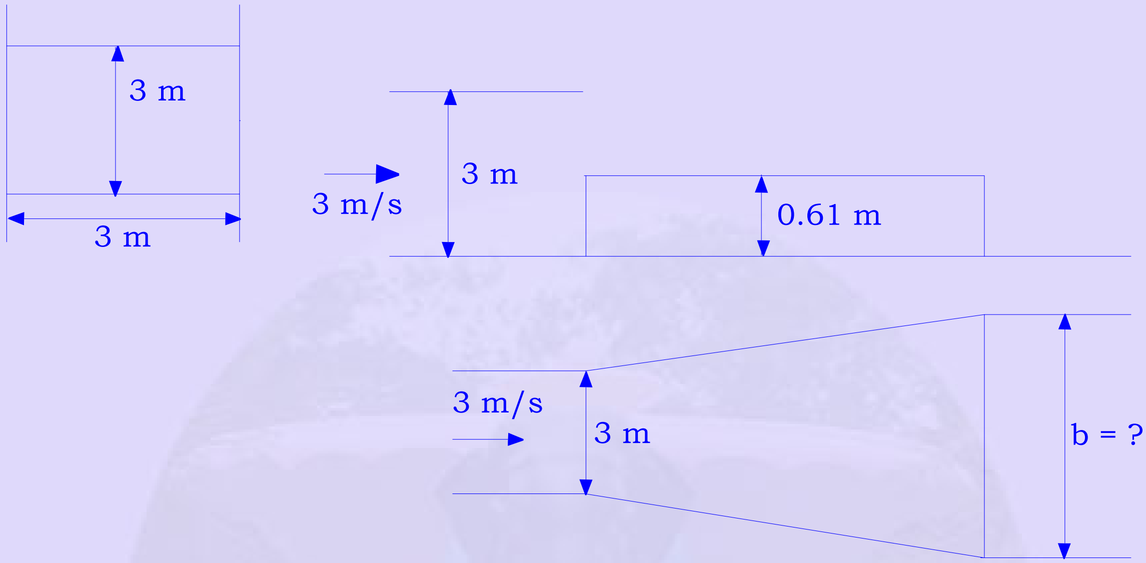
$$\therefore$$

$$\overline{V}_1 = 131.866\ \text{cm/s}$$

$$\boxed{Q = 6.026\ \text{l/s/cm}}$$

6. Water flows in a rectangular channel 3 m wide at a velocity of 3 m/s at a depth of 3 m. There is an upward step of 0.61 m. What expansion in width must take place simultaneously for this critical flow to be possible?

Solution



$$Q = 3*3*3 = 27 \ m^3/s$$

$$q = \frac{Q}{b} = \frac{27}{3} = 9.0 \ m^3/s/m$$

$$y_c = \left(\frac{q_c^2}{g}\right)^{1/3} = \left(\frac{9.0^2}{9.81}\right)^{1/3} = 2.021 \ m$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 3 + \frac{3^2}{19.2} = 3.46875 \ \frac{N\ m}{N}$$

$$E_1' = \frac{E_1}{y_c} = \frac{3 + \frac{3^2}{19.2}}{2.021} = 1.72$$

Downstream specific energy $E_2' = E_1' - \frac{\Delta Z}{y_c} = \frac{3.46875}{y_c} - \frac{0.61}{2.021} = 1.4145$

If the flow has to be critical

$E_2 = 3.46875 - 0.61 = 2.85875$ $\qquad E_2' = \frac{E_2}{y_{c2}} = 1.5$

$$y_{c2} = \frac{E_2}{1.5} = \frac{2.85875}{1.5} = 1.9058$$

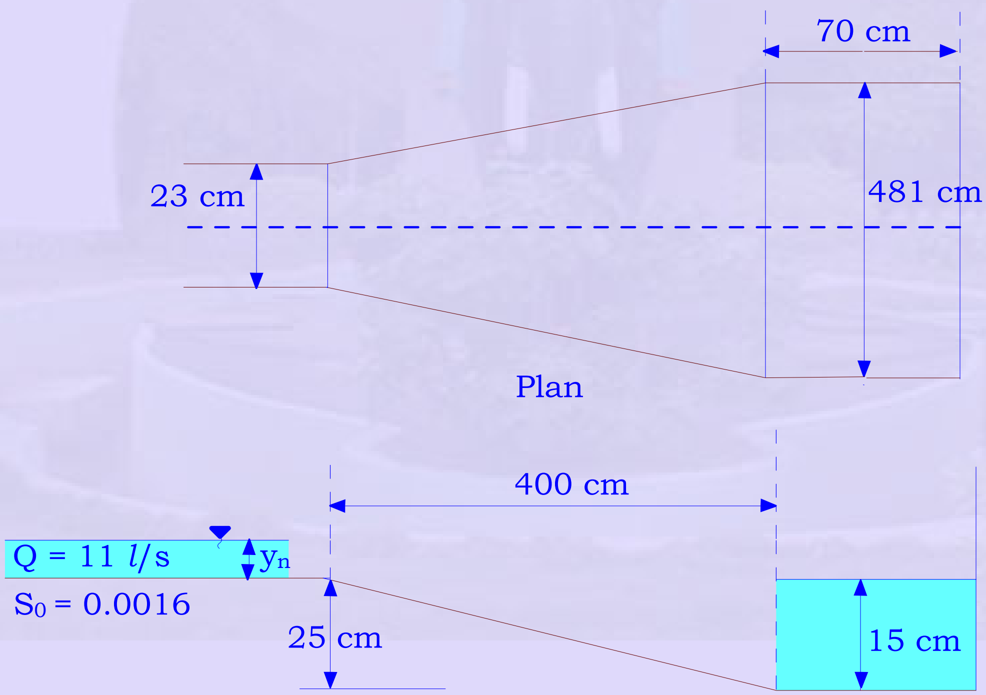
$$Q = b_2 y_{c2} V_{c2} = b_2 y_{c2} \sqrt{g y_{c2}}$$

$$\therefore\ b_2 = \frac{27}{1.9058\sqrt{9.81*1.9058}} = 3.2765 \text{ m}$$

$\therefore$ For critical flow to occur downstream width must be 3.2765 m.

Minimum expansion permitted is 0.2765 m in width

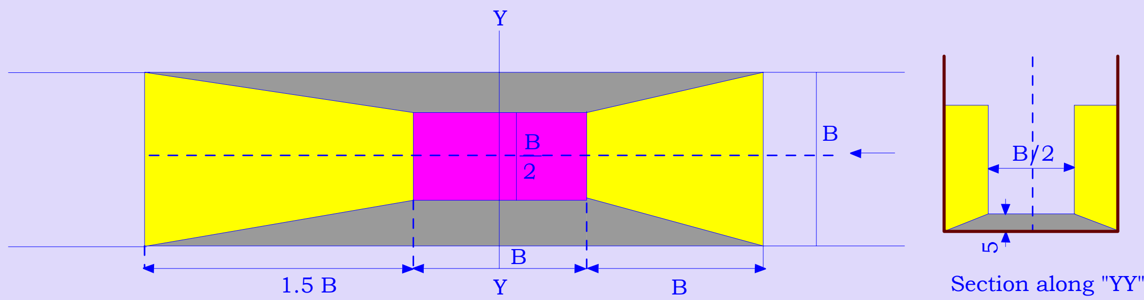
7. A rectangular channel of width 23 cm expands to 481 cms over a horizontal distance of 400 cm. The approach channel has a bed slope of 0.0016. At the junction bed drops by 25 cm over a length of 400 cm. The discharge is 11 l/s. The approach flow is uniform flow. In the downstream a minimum depth of 15 cm is sustained. A maximum water level of 40 cm is expected. Study the flow profiles for different downstream depths (between 15 cm to 40 cm). Locate the jump if it occurs.



Longitudinal sectional Elevation

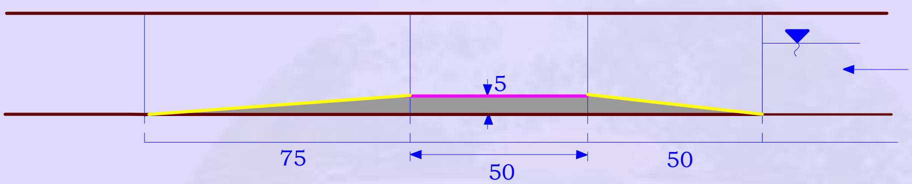
8. A transition is as shown in figure. Obtain the water surface profile if the width of the approaching channel is 50 cm. A discharge of 150 l/s is allowed into the channel at a depth of 35 cm. Downstream depth is controlled and a depth of 15 cm is maintained. Examine the possibility of a hydraulic jump after the

downstream after the transition and if the jump has to occur downstream of the transition, what necessary modifications are required.



Plan

Section along "YY"



Longitudinal Section along a Transition
(All dimensions are in cm)

9. A rectangular channel of 3.0 m width is narrowed down to 2.5 m by a contraction in a length of 20 m, built of straight walls and a horizontal bed. If the discharge is 3.5 m^3/s and the depth of flow is 1.50 m on the upstream side of the transition, determine the flow surface profile in the contraction (i) allowing no gradual hydraulic drop (ii) allowing a gradual hydraulic drop having its point of inflexion at the mid section of the contraction. Neglect frictional losses.