Lecture 8 - Spur gear design

CONTENTS

1. Surface durability –basic concepts
2. Surface failures
3. Buckingham contact stress equation
4. Contact stress –AGMA procedure
5. Surface fatigue strength – AGMA procedure
6. Permissible contact stress

SPUR GEAR - SURFACE DURABILITY BASIC CONCEPTS

Earlier various types of gear failures have been discussed in detail. Under contact conditions, gear teeth are subjected to Hertzian contact stresses and elasto-hydrodynamic lubrication. Excessive loading and lubrication breakdown can cause combinations of abrasion, pitting and scoring.

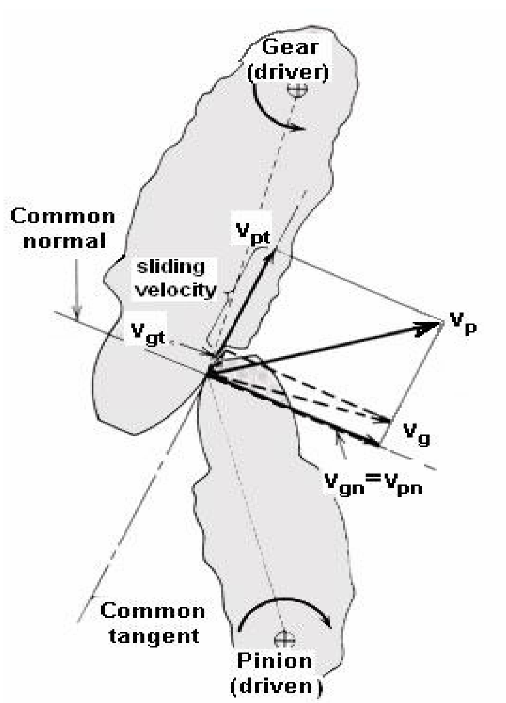


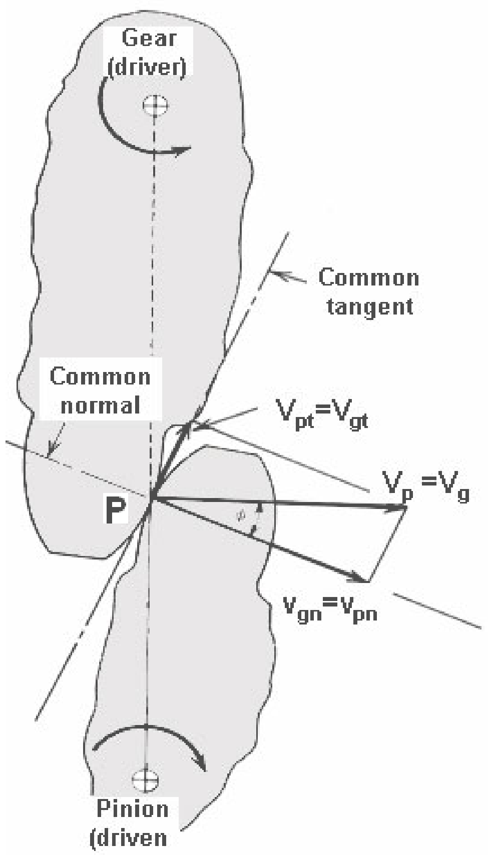
Figure shows the contact of a gear and pinion tooth at the tip. The Instantaneous contact point velocities of gear and pinion are vectorially denoted by Vg and Vp

a. The individual velocity can be resolved into two components normal and tangent to the tooth surface as V_{pn} and V_{pt} for the pinion and V_{gn} and V_{gt} for the gear respectively.

b. When the teeth do not separate or crush together, the components V_{pn} and V_{gn} normal to the surface must be the same.

c. Hence tangential velocity components of the surface are different.

d. The sliding velocity is the difference between V_{pt} and V_{gt}



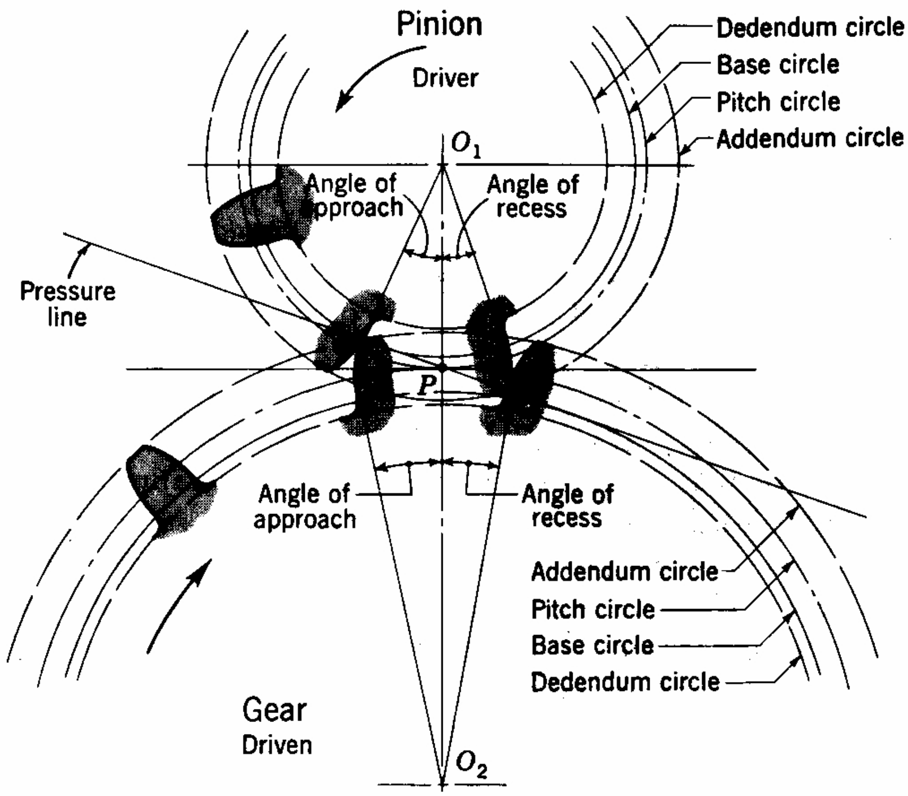
If the contact is at the pitch point P, the sliding velocity is zero; the tooth relative motion is pure rolling.

a) At all the other contact points, the relative motion is one of pure rolling and sliding.

b) The sliding velocity is directly proportional to the distance between the pitch point and the point of contact.

c) The maximum sliding velocity occurs with contacts at the tooth tips.

d) Gear teeth with longer addenda have higher sliding velocities than gears with shorter addenda.



The relative sliding velocity reverses the direction as a pair of teeth roll through the pitch point. During approach the sliding Friction forces tend to compress the teeth; during recess, Friction forces tend to elongate the teeth. Elongated teeth tend to give smoother action.

SPUR GEAR-SURFACE FAILURES

a. Gear teeth are also subjected to Hertz contact stresses,and the lubrication is often *elastohydrodynamic*.
b. Excessive loading and lubrication breakdown results in various combinations of abrasion, pitting and scoring.

1. Abrasive wear is caused
   a) by the presence of foreign particles, in gears that are not enclosed,
   b) in enclosed gears that were assembled with abrasive particles present,
   c) in gears lubricated by an oil supply with inadequate filtration.

2. Scoring:
   a) It occurs at high speeds when adequate lubrication is not provided by the elasto-hydrodynamic action.
   b) Lack of lubrication causes high sliding friction. High tooth loading and high sliding velocities that produce a high rate of heat in the localized contact region causes welding and tearing of surfaces apart.

c) Scoring can often be prevented by directing adequate flow of appropriate lubricant that maintains hydrodynamic lubrication.

d). Surface finish is also an important factor for scoring. Surface finish as fine as 0.5µm cla is desirable to avoid scoring.

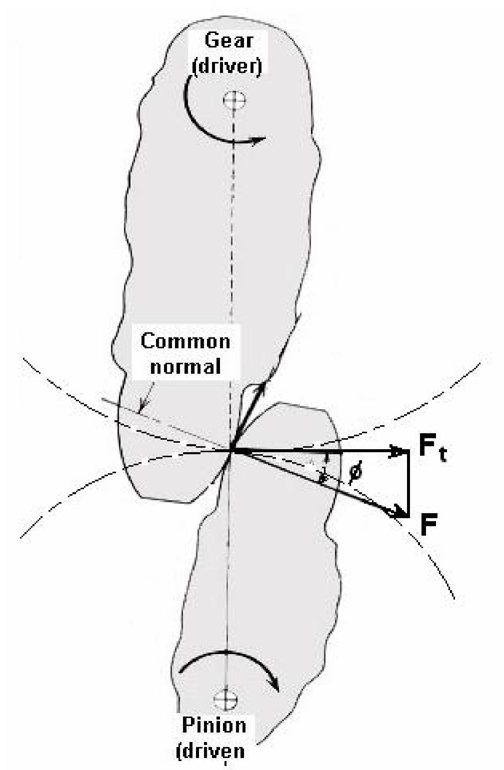
3. Pitting or surface fatigue failure:
Complex stresses within the contact zone cause surface and subsurface fatigue failures.
Pitting occurs only after a large a number of repeated loading mainly in the vicinity of the pitch line where the oil film breaks down because of zero sliding velocity.

SPUR GEAR – BUCKINGHAM CONTACT STRESS EQUATION

Buckingham adapted the Hertz contact stress equation for a pair of gear teeth. He treated a pair of gear teeth as two cylinders of radii equal to the radii of curvature of the mating involutes at the pitch point. From basic involute geometry, these radii are

$$R_1 = (d_1 \sin \text{Ø}) / 2 \ \ \& \ \ R_2 = (d_2 \sin \text{Ø}) / 2 \quad (1)$$

If the contact load $F = F_t / \cos\text{Ø}$, face width of pinion b ; contact radii of pinion and gear R_1 and R_2; their Poisson's ratios μ_1 and μ_2, and elastic modulii E_1 and E_2 respectively, then Hertzian contact stress under static condition is given by:



$$\sigma_H' = 0.564\sqrt{\frac{FE}{bR}} \qquad (2)$$

Where $F = F_t / \cos Ø$ (3)

$$\frac{1}{E} = \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2} \qquad (4)$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{d_1 \sin\varphi} + \frac{2}{d_2 \sin\varphi} \qquad (5)$$

Substituting the value of F_t, E and R from equation (3),(4) & (5) into (2) we get

$$\sigma_H' = 0.564\sqrt{\frac{F_t\left(\frac{2}{d_1 \sin\varphi} + \frac{2}{d_2 \sin\varphi}\right)}{b\cos\varphi\left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)}} \qquad (6)$$

It is seen from the above equation that

- Since contact area also increases with load, the contact stress increases only as the square root of load F_t
- Contact area increases with decrease of modulii of elasticity, E_1 and E_2.
- Larger gears have greater radii of curvature, hence lower stress.

Equation (6) can be rewritten by combining terms relating to the elastic properties of the material into single factor C_p given by:

$$C_p = 0.564\sqrt{\frac{1}{\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}}} \qquad (7)$$

The C_p value are given in Table 1.

Table 1. Elastic Coefficient C_p for Spur Gears, in $\sqrt{MPa}$

Pinion Material (μ = 0.3 in all cases)	Gear Material			
	Steel	Cast Iron	Al Bronze	Tin Bronze
Steel, E = 207 GPa	191	166	162	158
Cast Iron, E = 131 GPa	166	149	149	145
Al Bronze, E=121 GPa	162	149	145	141
Tin Bronze, E=110 GPa	158	145	141	137

- and combining terms relating to tooth shape into second factor, I, known as the geometry factor:

$$I = \frac{\sin\varphi\cos\varphi}{2}\,\frac{i}{i+1} \qquad (8)$$

- Where the speed ratio $i = d_2 / d_1$

The simplified contact stress equation now is:

$$\sigma_H' = C_p\sqrt{\frac{F_t}{b\,d_1\,I}} \qquad (9)$$

In this equation F_t is considered as static since the Hertz equation is derived for static loads.

Rearranging the terms

$$F_t = b\,d_1\,I\left(\frac{\sigma_H'}{C_p}\right)^2 \qquad (10)$$

If we substitute σ_H' by the permissible stress $[\sigma_H]$ for the material, then what we get is the tooth surface strength of the pinion F_{ts}.

$$F_{ts} = b\,d_1\,I\left(\frac{[\sigma_H]}{C_p}\right)^2 \qquad (11)$$

$$\boxed{F_{ts} > F_d \qquad (12)}$$

for safe operation of the gear from surface fatigue considerations. F_d is the Buckingham dynamic load on gear tooth. This approach gives quick results for preliminary design. This is the Buckingham design approach for wear strength.

SPUR GEAR –CONTACT STRESS AGMA

Introducing the factors K_v, K_o and K_m used in the bending fatigue analysis into the contact stress equation, the dynamic contact stress is obtained as σ_H :

$$\sigma_H = C_p \sqrt{\frac{F_t}{b\, d_1\, I} K_V K_o K_m} \qquad (13)$$

SPUR GEAR – SURFACE DURABILITY

K_v = Velocity or dynamic factor, indicates the severity of impact on successive pairs of teeth during engagement. This is a function of pitch line velocity and manufacturing accuracy. It is given by equation (14), (15) and (16).

$$K_v = \frac{6+V}{6} \qquad (14)$$

Equation (14) is used for cut or milled teeth or for gears not carefully generated.

$$K_v = \frac{50+(200V)^{0.5}}{50} \qquad (15)$$

Equation (15) is used for hobbed and shaped gears.

$$K_v = \left[\frac{78+(200V)^{0.5}}{78}\right]^{0.5} \qquad (16)$$

Equation (16) is used for high-precision shaved or ground teeth.

K_o = Overload factor which reflects the degree of non-uniformity of driving and load torques. It is given in Table 2

Table 2 -Overload factor K_o

	Driven Machinery		
Source of power	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

K_m = Load distribution factor which accounts for non uniform spread of the load across the face width. It depends on the accuracy of mounting, bearings, shaft deflection and accuracy of gears. Values are given in Table 3.

Table 3. Load distribution factor K_m

	Face width b (mm)			
Characteristics of Support	0 - 50	150	225	400 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across the full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists	Over 2.2	Over 2.2	Over 2.2	Over 2.2

SPUR GEAR – SURFACE FATIGUE STRENGTH (AGMA)

Surface fatigue strength of the material is given by:

$\sigma_{sf} = \sigma_{sf}' \, K_L \, K_r \, K_T$ (17)

Where

σ_{sf}' = surface fatigue strength of the material given in Table 4

K_L = Life factor given in Fig.1

K_R = Reliability factor , given in Table 5

Table 4. Surface fatigue strength σ_{sf}' (MPa) for metallic spur gears,(10^7 cycle life 99% reliability and Temperature <120° C)

Material	σ_{sf}' (MPa)
Steel	2·8 (Bhn) – 69 MPa
Nodular Iron	0.95 [2·8 (Bhn) – 69 MPa]
Cast Iron , grade 20	379
Cast Iron , grade 30	482
Cast Iron , grade 40	551
Tin Bronze, AGMA 2C (11% Sn)	207
Aluminium Bronze (ASTM B 148 – 52) (Alloy 9C – H.T.)	448

Fig. 1 Life Factor K_L

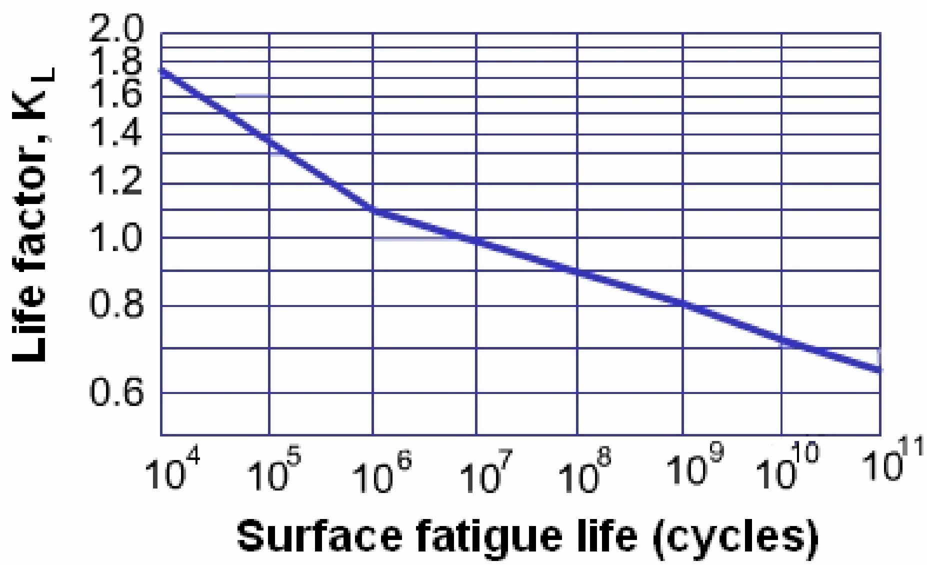


Table 5. Reliability factor K_R

Reliability (%)	K_R
50	1.25
99	1.00
99.9	0.80

K_T = temperature factor,
= 1 for $T \leq 120^oC$
based on Lubricant temperature.
Above 120^oC, it is less than 1 to be taken from AGMA standards.

SPUR GEAR – ALLOWABLE SURFACE FATIGUE STRESS (AGMA)

Allowable surface fatigue stress for design is given by

$$[\sigma_H] = \sigma_{Sf} / s \qquad (18)$$

Factor of safety s = 1.1 to 1.5
Hence Design equation is :

$$\boxed{\sigma_H \leq [\sigma_H]} \qquad (19)$$

GEAR MATERIALS

Gears are commonly made of cast iron, steel, bronze, phenolic resins, acetal, nylon or other plastics.
The selection of material depends on the type of loading and speed of operation, wear life, reliability and application.

Cast iron is the least expensive. ASTM / AGMA grade 20 is widely used. Grades 30,40,50, 60 are progressively stronger and more expensive.

CI gears have greater surface fatigue strength than bending fatigue strength. Better damping properties which enables them to run quietly than steel.

Nodular cast iron gears have higher bending strength together with good surface durability. These gears are now a days used in automobile cam shafts.

A good combination is often a steel pinion mated against cast iron gear.

Steel finds many applications since it combines both high strength and low cost. Plain carbon and alloy steel usage is quite common.

Through hardened plain carbon steel with 0.35 - 0.6% C are used when gears need hardness more than 250 to 350 Bhn. These gears need grinding to overcome heat treatment distortion.

When compactness, high impact strength and durability are needed as in automotive and mobile applications, alloy steels are used. These gears are surface or case-hardened by flame hardening, induction hardening, nitriding or case carburizing processes. Steels such as En 353, En36, En24, 17CrNiMo6 widely used for gears.

SPUR GEAR – GEAR MATERIALS

Bronzes are used when corrosion resistance, low friction and wear under high sliding velocity is needed as in worm-gear applications.
AGMA recommends Tin bronzes containing small % of Ni, Pb or Zn. The hardness may range from 70 to 85 Bhn.

Non metallic gears made of phenolic resin, acetal, nylon and other plastics are used for light load lubrication free quiet operation at reasonable cost. Mating gear in many such applications is made with steel. In order to accommodate high thermal expansion, plastic gears must have higher backlash and undergo stringent prototype testing.