5.6 Design for Torsion (Part II)

This section covers the following topics.

- Design of Transverse Reinforcement
- Detailing Requirements
- Design Steps

5.6.1 Design of Transverse Reinforcement

For the design of the transverse reinforcement, the capacities of concrete to resist the torsion and shear need to be determined. To consider the simultaneous occurrence of flexural and torsional shears, a linear interaction between the two is considered.

The capacity of concrete to resist torsion is reduced from T_c, the capacity under pure torsion. Similarly, the capacity of concrete to resist shear is reduced from V_c, the capacity in absence of torsion.

Capacity of Concrete under Pure Torsion

The capacity of concrete is determined based on the plastic theory for torsion. The capacity is equal to the torque generating the first torsional crack (T_{cr}). For a reinforced concrete beam, T_{cr} is estimated by equating the maximum torsional shear stress (τ_{max}) caused by T_{cr} to the tensile strength of concrete ($0.2\sqrt{f_{ck}}$). The estimated tensile strength is less than that under direct tension because the full section does not plastify as assumed in the plastic theory.

The estimate of the cracking torque (T_{cr}) for a rectangular section is given below.

$$T_{cr} \approx 0.2\sqrt{f_{ck}}\,\frac{b^2D}{2}\left(1-\frac{b}{3D}\right)$$

$$T_{cr} = 0.1\,b^2D\left(1-\frac{b}{3D}\right)\sqrt{f_{ck}} \qquad \textbf{(5-6.1)}$$

For flanged sections, the section is treated as a compound section. A compound section is a summation of rectangular sections.

The cracking torque is estimated as a summation of the capacities of the individual rectangular sections. Since the interaction between the rectangular sections is neglected in the summation, the estimate of the cracking torque is a lower bound estimate.

The following flanged section is shown as a compound section of five rectangles. For an individual rectangle, the short side is *b* and the long side is *D*.

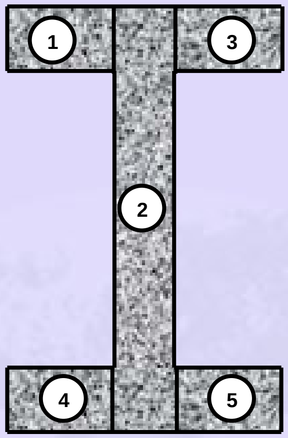


Figure 5-6.1 Flanged section as a compound section

The estimate of the cracking torque (T_{cr}) for a compound section is as follows.

$$T_{cr} = \sum 0.1 b^2 D \left(1 - \frac{b}{3D}\right)\sqrt{f_{ck}} \qquad \textbf{(5-6.2)}$$

Here, the summation is for the individual rectangles.

For a prestressed concrete beam, the strength of concrete is multiplied by the factor λ_p, which is a function of the average effective prestress (f_{cp}).

$$\lambda_p = \sqrt{1 + \frac{12 f_{cp}}{f_{ck}}} \qquad \textbf{(5-6.3)}$$

The value of f_{cp} is taken as positive (numeric value). It can be observed that the strength increases with prestress. The cracking torque (T_{cr}) and the capacity of concrete to resist torsion (T_c) for a prestressed concrete beam are thus estimated as follows.

$$T_c = T_{cr}$$

$$T_c = \sum 0.15 b^2 D \left(1 - \frac{b}{3D}\right)\lambda_p \sqrt{f_{ck}} \qquad \textbf{(5-6.4)}$$

In the previous expression,

b = breadth of the individual rectangle

D = depth of the individual rectangle.

Interaction of Shear and Torsion

In presence of flexural shear, the torsional capacity of concrete reduces. Similarly, in presence of torsion, the flexural shear capacity of concrete reduces. This is referred to as interaction of shear and torsion. The capacity of concrete under shear is explained in Section 5.2, Design for Shear (Part I). A linear interaction of the shear and torsion capacities of concrete is considered as shown in the following figure. In the horizontal axis, the shear demand is normalised with respect to the capacity of concrete under flexural shear. In the vertical axis, the torsional demand is normalised with respect to the capacity of concrete under pure torsion.

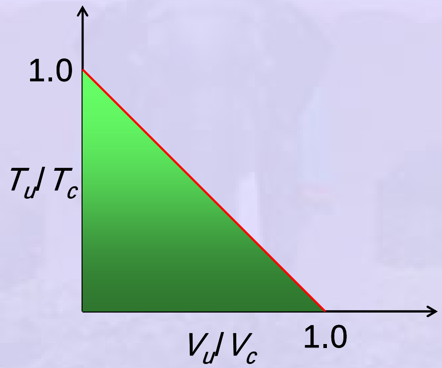


Figure 5-6.2 Interaction diagram for shear and torsion

The interaction equation is given as follows.

$$\frac{T_u}{T_c} + \frac{V_u}{V_c} = 1 \quad \textbf{(5-6.5)}$$

This is a linear interaction equation.

In the previous expression,

T_u = applied torsion at ultimate

V_u = applied shear at ultimate

T_c = capacity of concrete under pure torsion.

V_c = capacity of concrete under flexural shear.

Based on the interaction equation, the reduced capacity of concrete to resist torsion (T_{c1}) is given below.

$$T_{c1} = T_c\left(\frac{e}{e+e_c}\right) \leq T_u / 2 \qquad \textbf{(5-6.6)}$$

T_{c1} is limited to $T_u/2$ to restrict concrete reaching its capacity.

The parameter e is the ratio of torsion and shear demands at ultimate. The parameter e_c is the ratio of the corresponding concrete capacities.

$$e = T_u / V_u \qquad \textbf{(5-6.7)}$$

$$e_c = T_c / V_c \qquad \textbf{(5-6.8)}$$

The reduced capacity of concrete to resist shear is given below.

$$V_{c1} = V_c \frac{e_c}{e+e_c} \qquad \textbf{(5-6.9)}$$

Calculation of Transverse Reinforcement

The transverse reinforcement is provided in the form of **closed** stirrups enclosing the corner longitudinal bars. The amount (A_{sv}) is equal to the higher value determined from two expressions.

The first expression is based on the **Skew Bending Theory**.

$$A_{sv} = \frac{M_t s_v}{1.5 b_1 d_1 f_y} \qquad \textbf{(5-6.10)}$$

The notations are as follows.

b_1 = distance between the corner longitudinal bars along the short side

d_1 = distance between the corner longitudinal bars along the long side.

M_t = additional bending moment from torsion.

s_v = spacing of the stirrups

f_y = characteristic yield stress of the stirrups.

The dimensions b_1 and d_1 are shown in the following sketch.

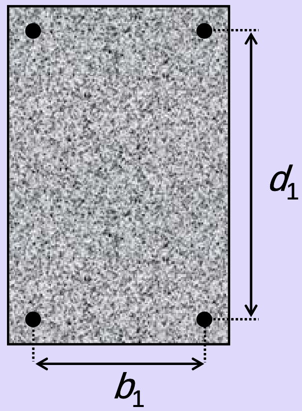


Figure 5-6.3 Dimensions between the corner bars

The second expression of A_{sv} is based on the concept of total shear.

$$A_{sv} = A_v + 2A_T \quad \textbf{(5-6.11)}$$

The first component A_v corresponds to the flexural shear to be carried by the stirrups. The second component A_t corresponds to the torsional shear to be carried by the stirrups. The factor 2 considers that the torsional shear is additive to flexural shear in both the legs.

The following sketch shows the addition of flexural and torsional shears for a hollow section.

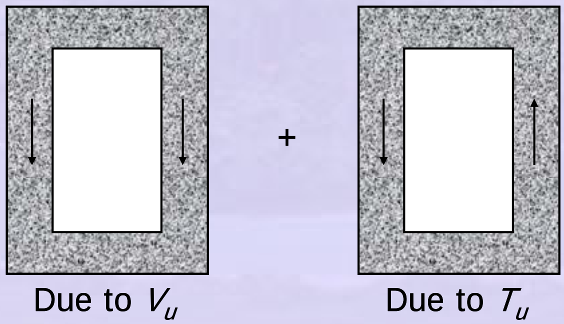


Figure 5-6.4 Distribution of flexural and torsional shears for a hollow section

The two shears are additive in the left web, whereas they are subtractive in the right web. Since, the stirrups have equal areas in the two legs, the torsional shear is considered additive to flexural shear in both the legs.

In solid sections, the two shears are not additive throughout the web. The flexural shear is distributed, whereas the torsional shear is restricted in the shear flow zone. Thus for solid sections, the expression of A_{sv} is conservative. The following sketch shows the addition of flexural and torsional shears for a solid section.

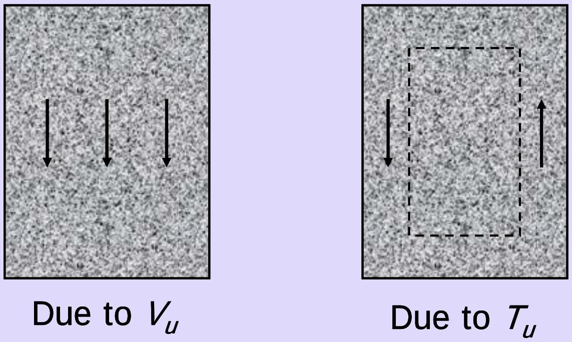


Figure 5-6.5 Distribution of flexural and torsional shears for a solid section

If the breadth of the web is large, the two shears can be designed separately. The stirrups for flexural shear can be distributed throughout the interior of the web. For torsional shear, closed stirrups can be provided in the peripheral shear flow zone.

The expressions of A_v and A_t are derived from the truss analogy for the ultimate limit state.

$$A_v = \frac{(V_u - V_{c1})s_v}{0.87f_y d_1} \quad \textbf{(5-6.12)}$$

$$A_T = \frac{(T_u - T_{c1})s_v}{0.87f_y b_1 d_1} \quad \textbf{(5-6.13)}$$

The minimum amount of transverse reinforcement is same as that for shear in absence of torsion.

$$\frac{A_{sv}}{bs_v} = \frac{0.4}{0.87f_y} \quad \textbf{(5-6.14)}$$

5.6.2 Detailing Requirements

The detailing requirements for torsional reinforcement in **Clause 22.5.5**, **IS: 1343 - 1980** are briefly mentioned.

1) There should be at least one longitudinal bar in each corner. The minimum diameter of the longitudinal bars is 12 mm.

When any side is larger than 450 mm, provide side face reinforcement ($A_{s,\ sf}$), as per the following.

Minimum amount $A_{s,sf,min}$ = 0.1% bD

Maximum spacing s_{max} = 300 mm or b, whichever is less.

This amount is sufficient to check thermal and shrinkage cracks.

2) The closed stirrups should be bent close to the tension and compression surfaces satisfying the minimum cover. The stirrups should be perpendicular to the axis of the beam. Closed stirrups should not be made of pairs of U-stirrups lapping one another. This is clarified in the following sketch.

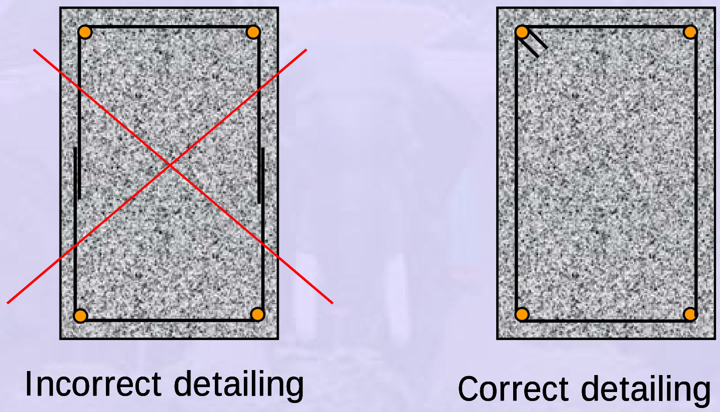


Figure 5-6.6 Detailing of closed stirrups

3) The maximum spacing is $(x_1 + y_1)/4$ or 200 mm, whichever is smaller. Here x_1 and y_1 are the short and long dimensions of the stirrups respectively.

4) Proper anchorage of stirrups as mentioned under detailing requirements of shear reinforcement. It is recommended to bend the ends of a stirrup by 135º and have 10 times the diameter of the bar (d_b) as extension beyond the bend. The following sketch clarifies the detailing of end hooks.

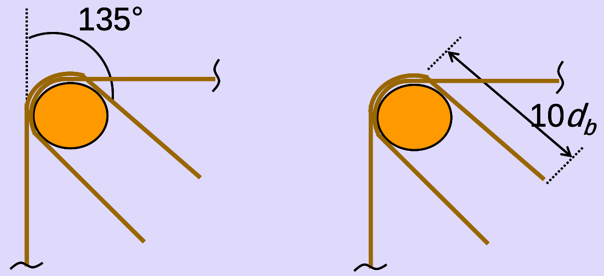


Figure 5-6.7 Detailing of end hooks for stirrups

5) The stirrups should be continued till a distance $h + b_w$ beyond the point at which it is no longer required. Here, h is the overall depth and b_w is the breadth of the web.

5.6.3 Design Steps

The following quantities are known at the selected section.

M_u = factored flexural moment

V_u = factored shear

T_u = factored torsional moment.

For gravity loads, these are calculated from the dead load and live load. The grades of concrete and steel are selected before design. As per **IS: 1343 - 1980**, the grade of steel for stirrups is limited to Fe 415.

For the design of longitudinal reinforcement, the following quantities are unknown.

The member cross-section.

M_{e1}, M_{e2}, M_{e3} = total equivalent flexural moment

A_p = amount of prestressing steel,

P_e = the effective prestress,

e = the eccentricity

A_s = area of longitudinal reinforcement

A_s' = area of longitudinal reinforcement in opposite face.

Prestressing steel A_p' may be provided in the opposite face.

For the design of stirrups, the following quantities are unknown.

V_{c1} = shear carried by concrete

T_{c1} = torsion carried by concrete

A_{sv} = total area of the legs of stirrups within a distance sv

s_v = spacing of stirrups.

The steps for designing longitudinal and transverse reinforcements for beams subjected to torsion are given.

1) Calculate M_u, V_u and T_u at a selected location. Select a suitable cross-section. For high value of T_u, as in bridges, a box section is preferred.

For longitudinal reinforcement

2a) Calculate M_{e1}.

2b) Design A_p and A_s. The design procedure involves preliminary design and final design, which are explained in the Section 4.2, Design of Sections for Flexure (Part I) and Section 4.3, Design of Sections for Flexure (Part II)

3a) Calculate M_{e2} if $M_u < M_t$.

3b) Design A_s'. The design procedure is similar for a reinforced concrete section. If A_p' is provided, the design is similar to a prestressed concrete section.

4a) Calculate M_{e3} if $M_u < M_t$.

4b) Check the adequacy of transverse bending based on the corner bars. If inadequate, design side face reinforcement ($A_{s,sf}$). $A_{s,sf}$ includes the corner bars. The design is similar to that for a reinforced concrete section.

For transverse reinforcement

5a) Calculate T_c, Eqn. **(5-6.4)**.

5b) Calculate V_c from the lower of V_{c0} and V_{cr}.

5c) Calculate e (if not calculated earlier) and e_c.

5d) Calculate T_{c1} and V_{c1}. Limit T_{c1} to $T_u/2$.

6) Calculate A_{sv} / s_v from the greater of the values given by Eqns. **(5-6.10)**, **(5-6.11)**, **(5-6.12)**, and **(5-6.13)**.

Compare the value with the minimum requirement Eqn. **(5-6.14)**.

7) Calculate maximum spacing and round it off to a multiple of 5 mm.

8) Calculate the size of the stirrups based on the amount required.

Repeat the calculations for other locations of the beam if the spacing of stirrups needs to be varied.

Example 5-6.1

Design a rectangular section to carry the following ultimate loads.

T_u = 44.5 kNm

M_u = 222.5 kNm (including an estimate of self-weight)

V_u = 89.0 kN.

The material properties are as follows.

f_{ck} = 35 N/mm^2

f_y = 250 N/mm^2

f_{pk} = 1720 N/mm^2

The prestressing is f_{pe} = 1035 N/mm^2.

Solution

1) Calculate M_{e1}.

Let D/b = 2

$$M_t = T_u\sqrt{1+\frac{2D}{b}}$$
$$= 44.5\sqrt{1+2\times 2}$$
$$= 99.5\text{ kNm}$$

$$M_{e1} = M_u + M_T$$
$$= 222.5 + 99.5$$
$$= 322.0\text{ kNm}$$

2) Select section. Design A_p and A_s.

Select

b = 250 mm

D = 500 mm

d = 450 mm.

Provide (2) 16 mm diameter corner bars. The flexural design results are as follows.

$$A_s = 2 \times 201 = 402 \text{ mm}^2.$$

Required amount of prestressing steel with $d_p = d = 450$ mm is $A_p = 484$ mm^2.

Provide 11 mm diameter strands with area = 70 mm^2.

Required number of strands = 484 / 70 = 6.8 → 7

Provided amount of prestressing steel

$$A_{p,prov} = 7 \times 70 = 490 \text{ mm}^2$$

3) Calculate M_{e2} .

Since $M_u > M_t$, design for M_{e2} is not required.

4) Calculate M_{e3} .

Since $M_u > M_t$, design for M_{e3} is not required.

5a) Calculate T_c .

$$f_{cp} = \frac{P_e}{A} = \frac{f_{pe} \times A_p}{bD} = \frac{1035 \times 490}{250 \times 500} = 4.06 \text{ N/mm}^2$$

$$\lambda_p = \sqrt{1 + \frac{12 f_{cp}}{f_{ck}}} = \sqrt{1 + \frac{12 \times 4.06}{35}} = 1.55$$

$f_{cp} < 0.3\, f_{ck}$. OK

$$T_c = 0.15b^2D\left(1 - \frac{b}{3D}\right)\lambda_p\sqrt{f_{ck}}$$

$$= 0.15 \times 250^2 \times 500 \times \left(1 - \frac{1}{3 \times 2}\right) \times 1.55\sqrt{35}\ \text{Nmm}$$

$$= 35.8\ \text{kNm}$$

5b) Calculate V_c from the lower of V_{co} and V_{cr} .

$$\frac{100A_p}{bd} = \frac{100 \times 490}{250 \times 450}$$

$$= 0.43$$

From Table 6, for M 35 concrete, τ_c = 0.46 N/mm^2.

$$f_{pt} = -\frac{P_e}{A} - \frac{P_e e^2}{I}$$

$$= -\frac{507{,}150}{125{,}000} - \frac{507{,}150 \times 200^2}{2.604 \times 10^9}$$

$$= -11.85\ \text{N/mm}^2$$

Here,

e = 450 – ½ 500

= 200 mm

I = 250 × 5003 / 12

= 2.604 × 109 mm^4.

$$M_0 = 0.8 f_{pt}\frac{I}{y}$$

$$= 0.8 \times 11.85 \times \frac{2.604 \times 10^9}{200}$$

$$= 123.43\ \text{kNm}$$

$$V_{cr} = \left(1 - 0.55\frac{f_{pe}}{f_{pk}}\right)\tau_c bd + M_0\frac{V_u}{M_u}$$

$$= (1 - 0.55 \times 0.6) \times \frac{0.46 \times 250 \times 450}{10^3} + 123.43 \times \frac{89}{222.5}$$

$$= 84.0\ \text{kN}$$

Here,

f_{pe}/f_{pk} = 1035 / 1720

= 0.6.

$$V_{co} = 0.67bD\sqrt{f_t^2 + 0.8f_{cp}f_t}$$
$$= 0.67 \times 250 \times 500\sqrt{1.42^2 + 0.8 \times 4.06 \times 1.42}$$
$$= 215.6 \text{ kN}$$

Here,

$$f_t = 0.24\sqrt{35}$$
$$= 1.42 \text{ N/mm}^2$$

$\therefore$ $V_c = V_{cr} = 84.0$ kN

5c) Calculate e and e_c .

$$e = \frac{T_u}{V_u} = \frac{44.5}{89.0} = 0.50 \text{ m}$$

$$e_c = \frac{T_c}{V_c} = \frac{35.8}{84.0} = 0.43 \text{ m}$$

5d) Calculate T_{c1} and V_{c1}.

$$T_{c1} = T_c \frac{e}{e + e_c}$$
$$= 35.82 \times \frac{0.50}{0.50 + 0.43}$$
$$= 19.26 \text{ kNm}$$

$$V_{c1} = V_c \frac{e_c}{e + e_c}$$
$$= 84.0 \times \frac{0.43}{0.50 + 0.43}$$
$$= 38.84 \text{ kN}$$

$$T_{c1} < \frac{T_u}{2} \text{ OK.}$$

6) Calculate A_{sv} / s_v

$$\frac{A_{sv}}{s_v} = \frac{M_t}{1.5b_1d_1f_y}$$
$$= \frac{99.5 \times 10^6}{1.5 \times 200 \times 400 \times 250}$$
$$= 3.3 \text{ mm}^2\text{/mm}$$

Estimated values

b_1 = 250 – 50

= 200 mm

d_1 = 500 – 100

= 400 mm.

$$\frac{A_v}{s_v} = \frac{V_u - V_{c1}}{0.87 f_y d_1}$$

$$= \frac{(89.0 - 38.8)\times 10^3}{0.87\times 250\times 400}$$

$$= 0.58\ \text{mm}^2/\text{mm}$$

$$\frac{A_T}{s_v} = \frac{T_u - T_{c1}}{0.87 f_y b_1 d_1}$$

$$= \frac{(44.5 - 19.26)\times 10^6}{0.87\times 250\times 200\times 400}$$

$$= 1.45\ \text{mm}^2/\text{mm}$$

$$\frac{A_{sv}}{s_v} = \frac{A_V}{s_v} + 2\times\frac{A_T}{s_v}$$

$$= 0.58 + 2\times 1.45$$

$$= 3.48\ \text{mm}^2/\text{mm}$$

Minimum amount of stirrups

$$\frac{A_{sv}}{bs_v} = \frac{0.4}{0.87 f_y}$$

$$\frac{A_{sv}}{s_v} = \frac{0.4\times 250}{0.87\times 250}$$

$$= 0.46\ \text{mm}^2/\text{mm}$$

Select

A_{sv} / s_v = 3.48 mm^2/mm.

7) Calculate maximum spacing

$$s_v \le \frac{x_1 + y_1}{4}$$
$$\le \frac{204 + 422}{4}$$
$$\le 156 \text{ mm}$$

Estimated values

x_1 = 250 – 46

= 204 mm

y_1 = 500 – 78

= 422 mm.

The other values of s_v do not govern.

8) Calculate the size of the stirrups

Select 2 legs of 12 mm diameter stirrups.

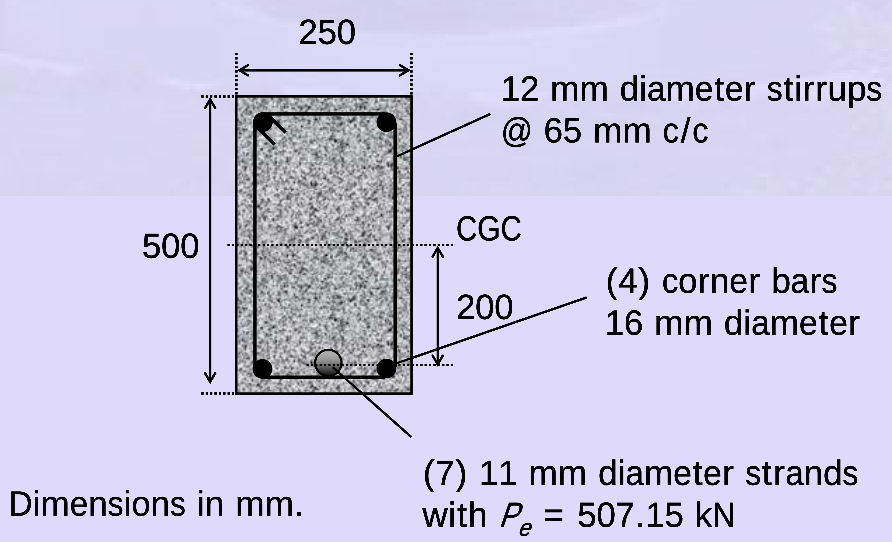
A_{sv} = 2 × 113

= 226 mm^2

$$s_v = \frac{226}{3.48}$$
$$= 65 \text{ mm}$$

The spacing can be increased by bundling the stirrup bars.

Designed section



As D > 450 mm, side face reinforcement is required.