

8.1 Cantilever Beams

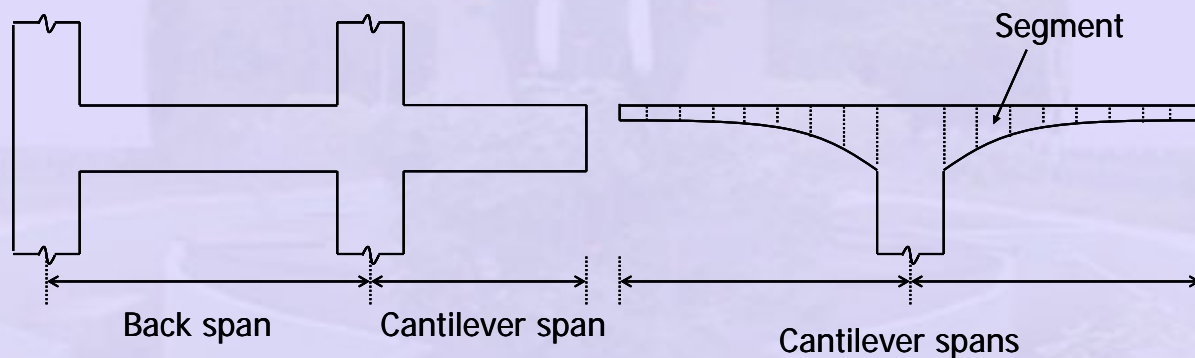
This section covers the following topics.

- Analysis
- Determination of Limiting Zone
- Tendon Profile

Introduction

Prestressed cantilever beams are present in buildings and bridges. Usually, the cantilever is provided with a back span (anchor span) to reduce the torsion in the supporting member. In a building, the cantilever can be an extension of a continuous beam. In a bridge, the cantilever is a part of the “balanced cantilever” girder. The construction can be in-situ, where the concrete is cast in formwork that is temporarily supported. Else, a segmental construction can be adopted, wherein slices of the girder are placed and temporarily prestressed.

The following figure illustrates the cantilevers in buildings and bridges schematically.



(a) Cantilever in a building frame

(b) Cantilevers in a bridge girder

Figure 8-1.1 Cantilevers in buildings and bridges



Figure 8-1.2 Segmental construction, Bridge over Hooghly, West Bengal (Courtesy: L & T Ramboll)

The analysis of a section of a prestressed cantilever for flexural stresses is similar to that for a simply supported beam. The difference is that for gravity loads, the bending moment in cantilever is negative, that is compression is generated at the bottom. Thus, the CGS is placed above the CGC and the eccentricity e is negative.

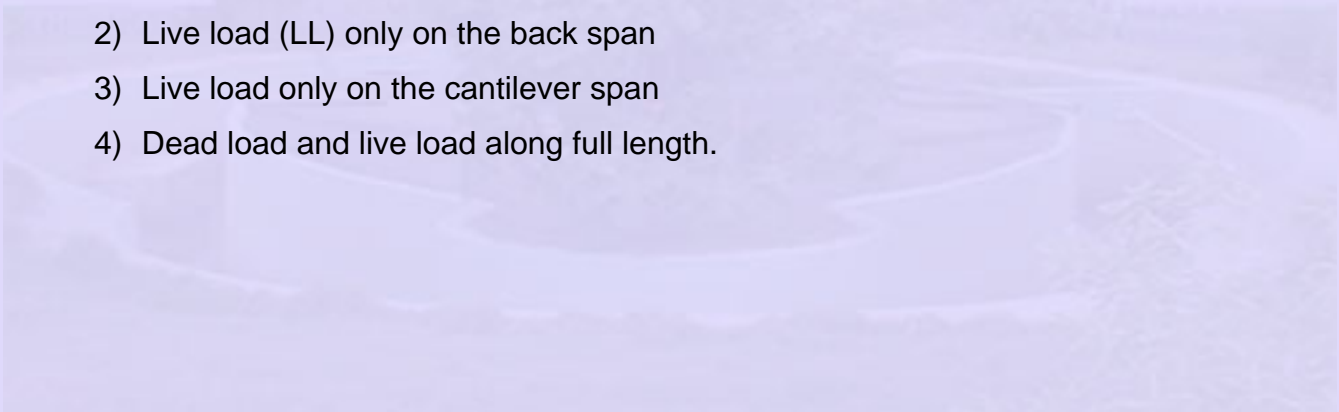
The following aspects need to be considered in the analysis and design of a prestressed cantilever beam.

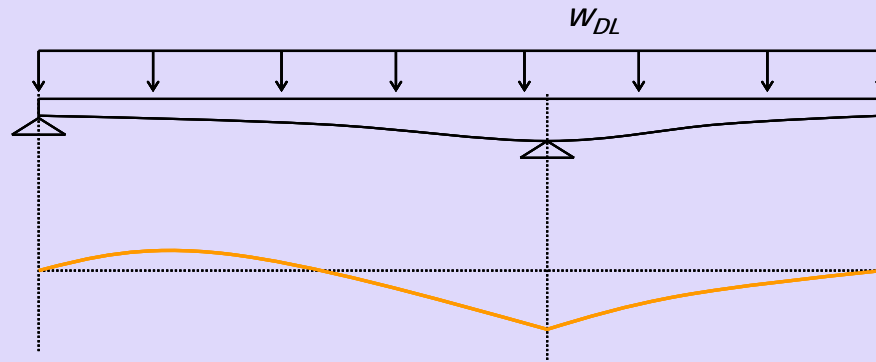
- 1) Certain portions of the back span are subjected to both positive and negative moments. Hence, there will be two design moments at service loads.
- 2) The beam may be subjected to partial loading and point loading.
- 3) The sequence of loading is important to design the prestressing force.
- 4) High values of moment and shear occur simultaneously near the support.

8.1.1 Analysis

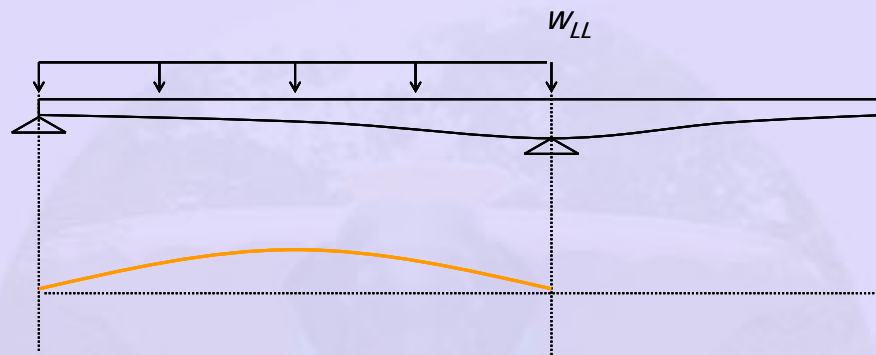
The analysis of a cantilever beam with a back span, is illustrated to highlight the aspects stated earlier. The bending moment diagrams for the following load cases are shown schematically in the following figure.

- 1) Dead load (DL)
- 2) Live load (LL) only on the back span
- 3) Live load only on the cantilever span
- 4) Dead load and live load along full length.

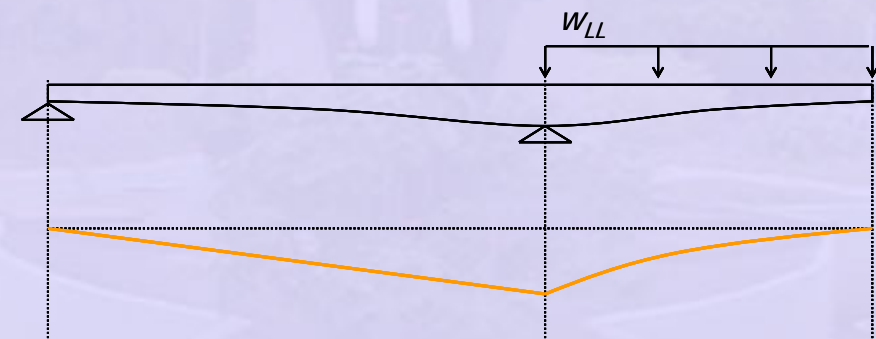




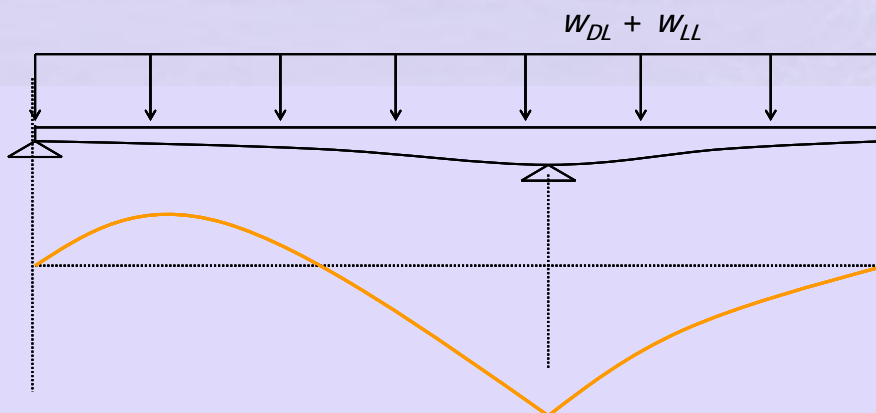
Moment diagram for DL



Moment diagram for LL on back span



Moment diagram for LL on cantilever span



Moment diagram for DL + LL along full length

Figure 8-1.3 Moment diagrams for dead and live loads

The envelop moment diagrams are calculated from the analysis of each load case and their combinations. In the following envelop moment diagrams, M_{max} and M_{min} represent the highest and lowest values (algebraic values with sign) of the moments at a section, respectively. Note that certain portions of the beam are subjected to both positive and negative moments.

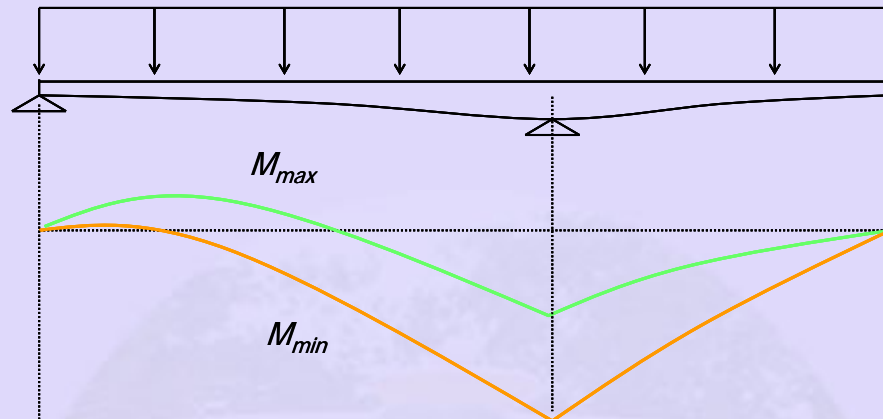


Figure 8-1.4 Envelop moment diagrams for dead and live loads

For moving point loads as in bridges, first the **influence line diagram** is drawn. The influence line diagram shows the variation of the moment or shear for a particular location in the girder, due to the variation of the position of a unit point load. The vehicle load is placed based on the influence line diagram to get the worst effect.

8.1.2 Determination of Limiting Zone

The limiting zone of placing the CGS of the tendons is helpful in selecting a tendon profile. Note that for a non-prismatic section, the section properties such as A , I , k_t , k_b , r change with position along the length.

The limiting zone was explained for a simply supported beam in Section 4.4, Design of Sections for Flexure (Part III). Here the concept and the equations are first reviewed for a simply supported beam with positive moment.

For full prestressed members (Type 1), tension is not allowed under service conditions. If tension is also not allowed at transfer, the compression in concrete (C) always lies within the kern zone. The limiting zone is defined as the zone for placing the CGS of

the tendons such that C always lies within the kern zone. Also, the maximum compressive stresses at transfer and service should be within the allowable values. For limited prestressed members (Type 2 and Type 3), tension is allowed at transfer and under service conditions. The limiting zone is defined as the zone for placing the CGS such that the tensile stresses in the extreme edges are within the allowable values. Also, the maximum compressive stresses at transfer and service should be within the allowable values.

The limiting zone is determined from the maximum or minimum eccentricities of the CGS along the beam corresponding to the extreme positions of C . Note that, the limiting zone is related with the CGS of the tendons. Individual tendons may lie outside the limiting zone.

For a simply supported beam, the maximum eccentricity (e_{max}) at any section corresponds to the lowest possible location of C at transfer, that generates allowable tensile stress at the top of the section. The maximum compressive stress at the bottom should be within the allowable value. The minimum eccentricity (e_{min}) at any section corresponds to the highest possible location of C at service, that generates allowable tensile stress at the bottom of the section. The maximum compressive stress at the top should be within the allowable value.

The values of e_{max} and e_{min} can be determined by equating the stresses at the edges of concrete with the allowable values. Else, explicit expressions of e_{max} and e_{min} can be developed. The following material gives the expressions of e_{max} and e_{min} for Type 1 and Type 2 sections. The values of e_{max} and e_{min} can be determined at regular intervals along the length of the beam. The zone between the loci of e_{max} and e_{min} is the limiting zone of the section for placing the CGS. The equations are summarised

Type 1 Section

Based on the stress at the top at transfer, the following expression of e_{max} is derived.

$$e_{max} - k_b = \frac{M_{sw}}{P_0}$$

$$\text{or, } e_{max} = \frac{M_{sw}}{P_0} + k_b \quad (8-1.1)$$

Also, the stress at the bottom should be less than the allowable value at transfer. Based on the stress at the bottom at service, the following expression of e_{min} is derived.

$$e_{min} + k_t = \frac{M_T}{P_e}$$

$$\text{or, } e_{min} = \frac{M_T}{P_e} - k_t \quad (8-1.2)$$

Also, the stress at the top should be less than the allowable value at service. If for a particular section e_{min} is negative, it implies that the CGS can be placed above the CGC. This happens near the supports.

Type 2 Section

The corresponding equations for a Type 2 section are given below.

$$e_{max} - k_b = \frac{M_{sw} + f_{ct,all} Ak_b}{P_0}$$

$$\text{or, } e_{max} = \frac{M_{sw} + f_{ct,all} Ak_b}{P_0} + k_b \quad (8-1.3)$$

$$e_{min} + k_t = \frac{M_T - f_{ct,all} Ak_t}{P_e}$$

$$\text{or, } e_{min} = \frac{M_T - f_{ct,all} Ak_t}{P_e} - k_t \quad (8-1.4)$$

In a simply supported beam, the external moments are always positive. The minimum moment is due to self weight. The maximum moment is under service loads. For cantilever beams, the minimum external moment need not be at transfer, when the moment is due to self weight (M_{SW}). Also, under service loads there are two moments M_{min} and M_{max} at a location, obtained from the envelop moment diagrams.

The maximum and minimum eccentricities e_{max} and e_{min} at a particular location are first determined for service loads from M_{min} and M_{max} , respectively, at that location and the effective prestress P_e . Next, another set of e_{max} and e_{min} are calculated at transfer from M_{SW} and the prestress P_0 . The final e_{max} is the lower of the two values calculated at service and at transfer. Similarly, the final e_{min} is the higher of the two values calculated at service and at transfer.

The expressions of e_{max} and e_{min} for the simply supported beam were developed for positive moments. For a cantilever, corresponding to a negative moment, the eccentricity implies that the CGS is located above CGC. The expressions for a cantilever are given below.

Type 1 section

At Service

$$e_{max} = \frac{M_{min}}{P_e} + k_b \quad (8-1.5)$$

$$e_{min} = \frac{M_{max}}{P_e} - k_t \quad (8-1.6)$$

At Transfer

$$e_{max} = \frac{M_{sw}}{P_0} + k_b \quad (8-1.7)$$

$$e_{min} = \frac{M_{sw}}{P_0} - k_t \quad (8-1.8)$$

Type 2 section

At Service

$$e_{max} = \frac{M_{min} + f_{ct,all} Ak_b}{P_e} + k_b \quad (8-1.9)$$

$$e_{min} = \frac{M_{max} - f_{ct,all} Ak_t}{P_e} + k_t \quad (8-1.10)$$

At Transfer

$$e_{max} = \frac{M_{sw} + f_{ct,all} Ak_b}{P_0} + k_b \quad (8-1.11)$$

$$e_{min} = \frac{M_{sw} - f_{ct,all} Ak_t}{P_0} - k_t \quad (8-1.12)$$

A typical limiting zone is shown in the following figure.

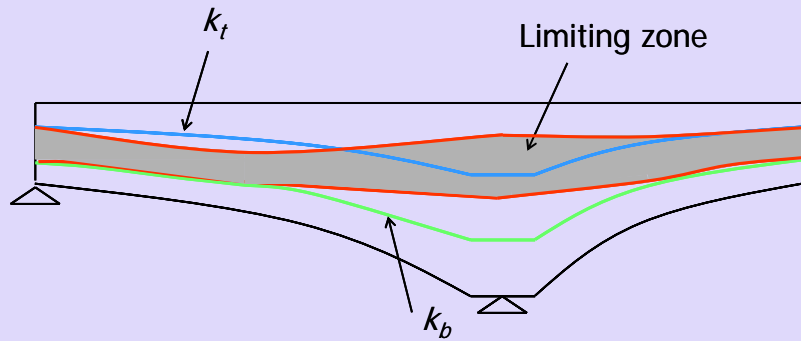
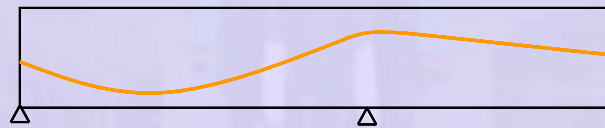


Figure 8-1.5 Limiting zone for a non-prismatic beam

8.1.3 Tendon Profile

The tendon profiles for a few beams with cantilever spans are shown schematically in the following figures. The vertical scale is enlarged to show the location of the CGS with respect to CGC.



a) Uniform cross section



b) Varying cross section in the cantilever span

Figure 8-1.6 Beams with single cantilever span

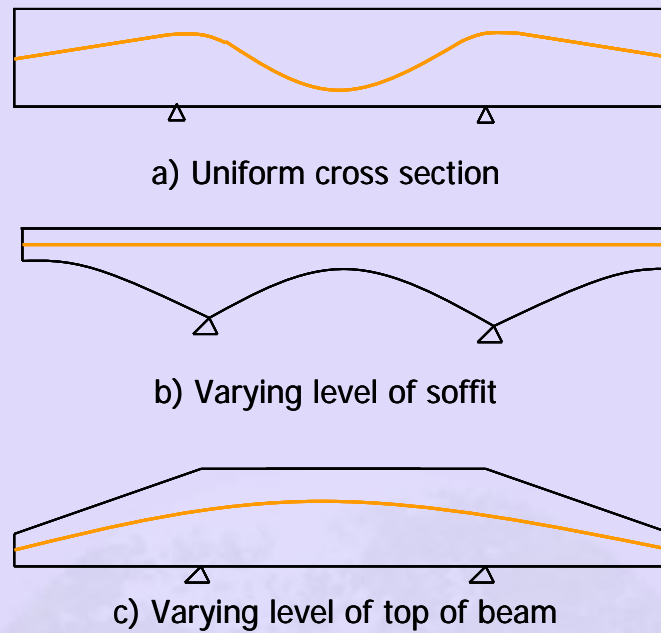


Figure 8-1.7 Beams with double cantilever spans

For a prismatic beam with uniform cross section along the length, the tendon profile is similar to the moment diagram under uniform load. Thus for regions of negative moment, the CGS is located above the CGC. Since there cannot be a sharp kink in the tendons and, the supports are not true point supports, the profile is shown curved at the right support.

For a beam with varying depth, the tendon profile can be adjusted (within e_{max} and e_{min}) to be straight for convenience of layout of the tendons.