8.3 Continuous Beams (Part II)

This section covers the following topics.

- Principle of Linear Transformation
- Concordant Tendon Profile
- Tendon Profiles
- Partially Continuous Beams
- Analysis for Ultimate Strength
- Moment Redistribution

Introduction

Before the discussion on the tendon profile (profile of the CGS), the following concepts are introduced.

1) Principle of linear transformation
2) Concordant tendon profile.

8.3.1 Principle of Linear Transformation

When the profile of the CGS is moved over the interior supports of a continuous beam without changing the intrinsic shape of the profile within each individual span, the profile is said to be linearly transformed. In a linear transformation, the curvatures remain constant and the locations of bends remain unchanged.

The following sketch explains the concept of linear transformation of the profile of the CGS.

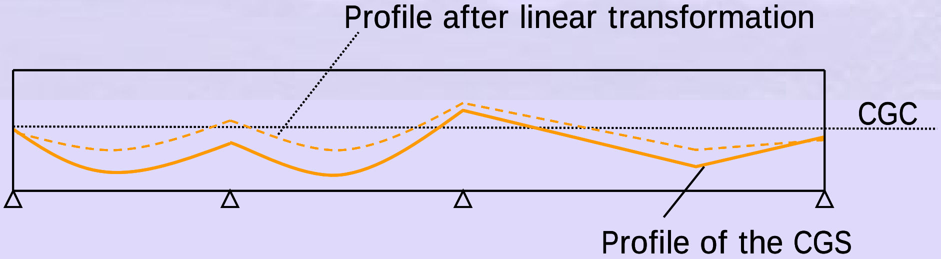


Figure 8-3.1 Linear transformation of the profile of the CGS

Linear transformation cannot involve the movement of the CGS at the ends of a beam or at the support of a cantilever.

Theorem

In a continuous beam, a profile of the CGS can be linearly transformed without changing the position of the resultant pressure line. This theorem can be proved based on the requirement that the curvature of the profile of the CGS remains constant under linear transformation. The following sketch explains that the pressure line remains constant for linearly transformed profiles of the CGS.

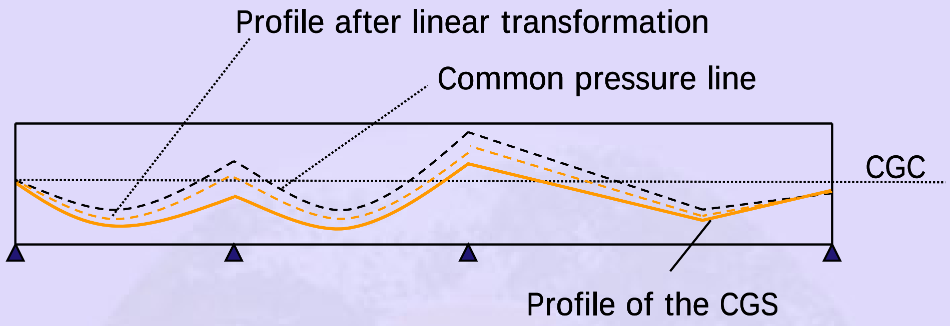


Figure 8-3.2 Pressure line for linearly transformed profiles of the CGS

8.3.2 Concordant Tendon Profile

A concordant tendon profile in a continuous beam is a profile of the CGS which produces a pressure line coincident with the profile itself. A concordant tendon profile does not produce reactions at the supports or secondary moments in the spans. The upward and downward equivalent loads balance each other.

The following sketch shows a concordant tendon profile which is coincident with the pressure line.

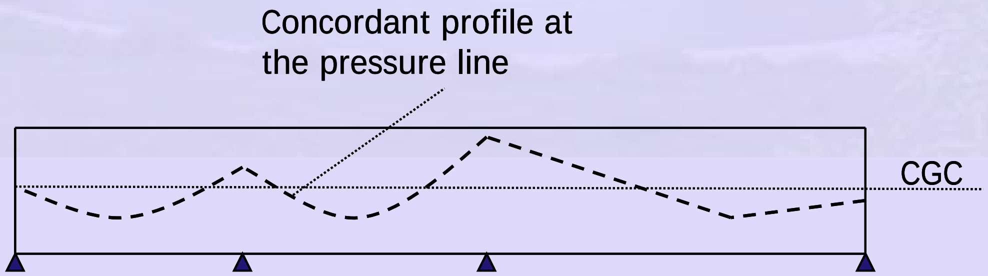


Figure 8-3.3 Concordant profile

The advantage of a concordant cable profile is that the calculations become simpler.

1) There is no secondary moment in the spans due to the prestress. The M_2 diagram coincides with the M_1 diagram.
2) The pressure line due to the prestress coincides with the cable profile. The shift of the pressure line due to external loads can be measured from the profile directly.

A concordant profile can be developed from the moment diagram due to external loads for a certain load combination using the following theorem.

Theorem

Every real moment diagram for a continuous beam on non-settling supports produced by any combination of external loads, whether transverse loads or moments, plotted to any scale, is one location for a concordant tendon in that beam.

The theorem can be proved based on the condition of no deflection at the supports due to external loads. Also, for a concordant profile since there is no reaction at any support, there is no possibility of deflections at the supports. Thus, it is easy to obtain a concordant profile from the moment diagram of the external loads for a certain load combination, drawn to a certain scale. The following figure shows the steps of the development of concordant profile from the moment diagram.

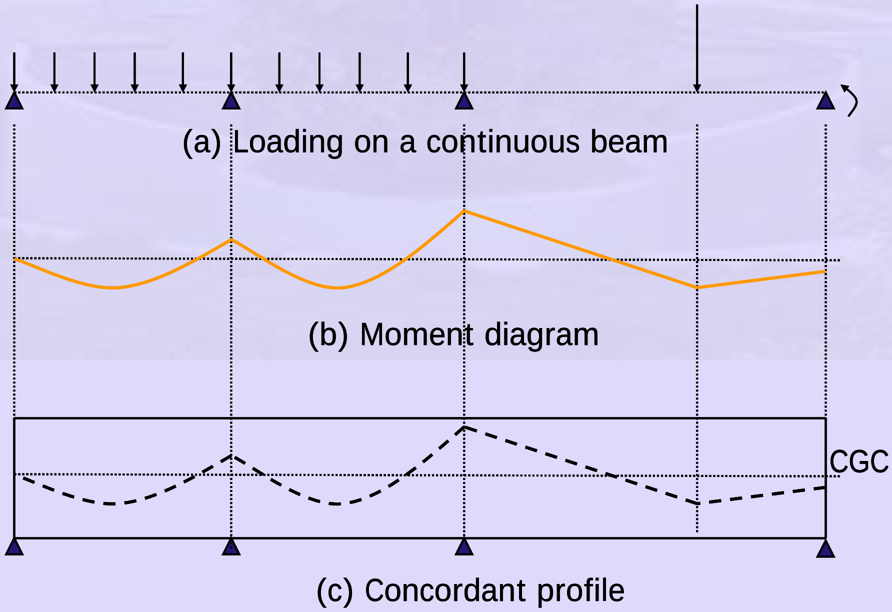


Figure 8-3.4 Development of concordant profile

Discussion

The computation of the concordant profile helps in the layout of the tendon profile. The tendon profile need not be designed to be a concordant profile. It should be such that the stresses in concrete at transfer and at service are within the allowable values. If a concordant profile is selected then the calculations become simpler.

8.3.3 Tendon Profiles

The steps of selecting a tendon profile (profile of the CGS) are based on trials. The steps are as follows.

1) Assume the section of the beam for calculating self weight. For the preliminary design, the type and depth (h) of the section can be selected based on architectural requirement and deflection criteria.
2) Calculate the moment due to self weight (M_{sw}) and the maximum moment (M_{max}) and minimum moment (M_{min}) along the length of the beam (envelop moment diagrams) due to the external loads, including self weight.
3) Compute the required P_e based on the values of M_{max} and M_{min}, at the critical locations, similar to the calculations for a simply supported beam. Revise the section if necessary. If M_{sw} is large,

$$P_e = M_T / z \quad \textbf{(8-3.1)}$$

$$z \approx 0.65h \quad \textbf{(8-3.2)}$$

Here,

M_T = M_{max} or M_{min}

z = estimated lever arm.

4) Considering $f_{pe} = 0.7f_{pk}$, calculate area of prestressing steel $A_p = P_e / f_{pe}$.
5) Check the area of the cross-section (A) based on $A = P_e /(0.5f_{cc,all})$.
6) Calculate the kern distances k_b and k_t, and the maximum and minimum eccentricities (e_{max} and e_{min}) along the length. The zone between e_{max} and e_{min} along the length of the beam is the limiting zone. The equations of e_{max} and e_{min} are same as that for a simply supported beam.

The value of P_0 can be estimated from P_i as follows.

a. 90% of the initial applied prestress (P_i) for pre-tensioned members.

b. Equal to P_i for post-tensioned members.

The value of P_i can be estimated as follows.

$$P_i = A_p\,(0.8f_{pk}) \qquad \textbf{(8-3.3)}$$

$$A_p = P_e / 0.7f_{pk} \qquad \textbf{(8-3.4)}$$

7) Select a trial profile of the CGS within the limiting zone. If the profile is a concordant profile, the pressure line due to prestress coincides with the profile of the CGS.

Calculate the shift in the pressure line due to external loads. For a Type 1 member, if the final pressure line lies within the kern zone, then the solution is acceptable. If final pressure line lies outside the kern zone, try another profile.

For Type 2 and Type 3 members, if the final pressure line lies within a zone such that the stresses at the edges are within the allowable values, then the solution is acceptable. If final pressure line lies outside the zone, try another profile.

8) Linearly transform the profile of the CGS to satisfy the cover requirements and the convenience of prestressing.

For a prismatic beam with uniform cross section along the length, the tendon profile can be selected similar to the moment diagram under uniform load. Since there cannot be a sharp kink in the tendons and the supports are not true point supports, the profile needs to be curved at an intermediate support. For a beam with varying depth, the tendon profile can be adjusted (within the limiting zone) to be relatively straight for convenience of layout of the tendons and reduction of losses due to friction. The tendons can be of segments of single curvature to reduce frictional losses.

The following sketches show the profiles of the CGS for common continuous beams.

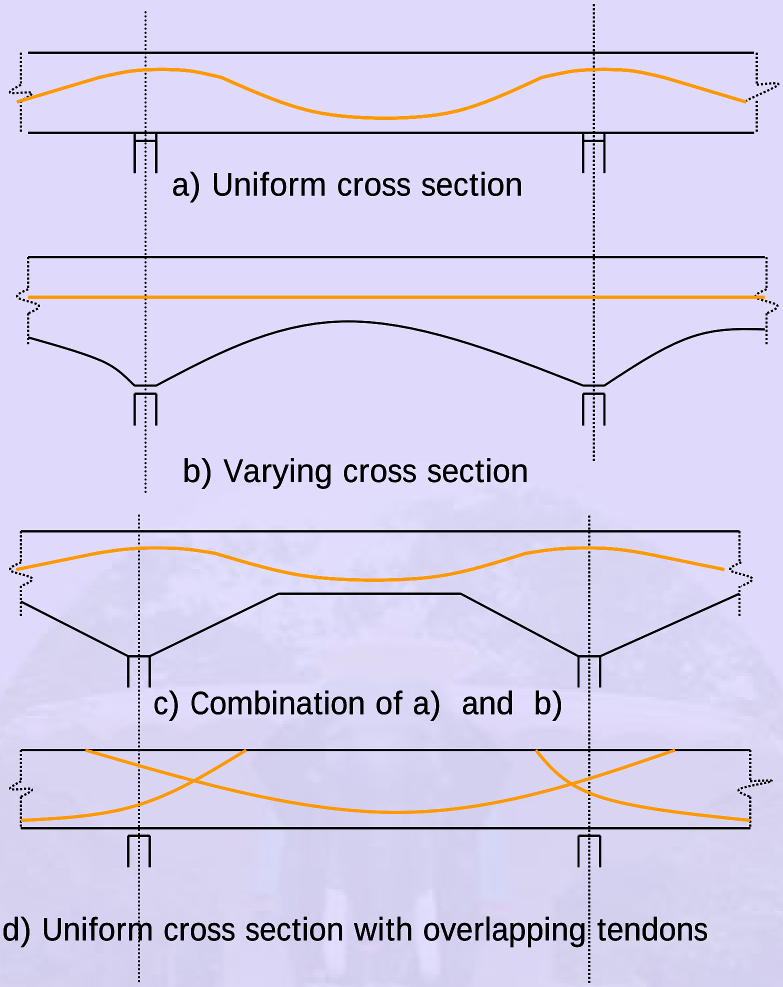


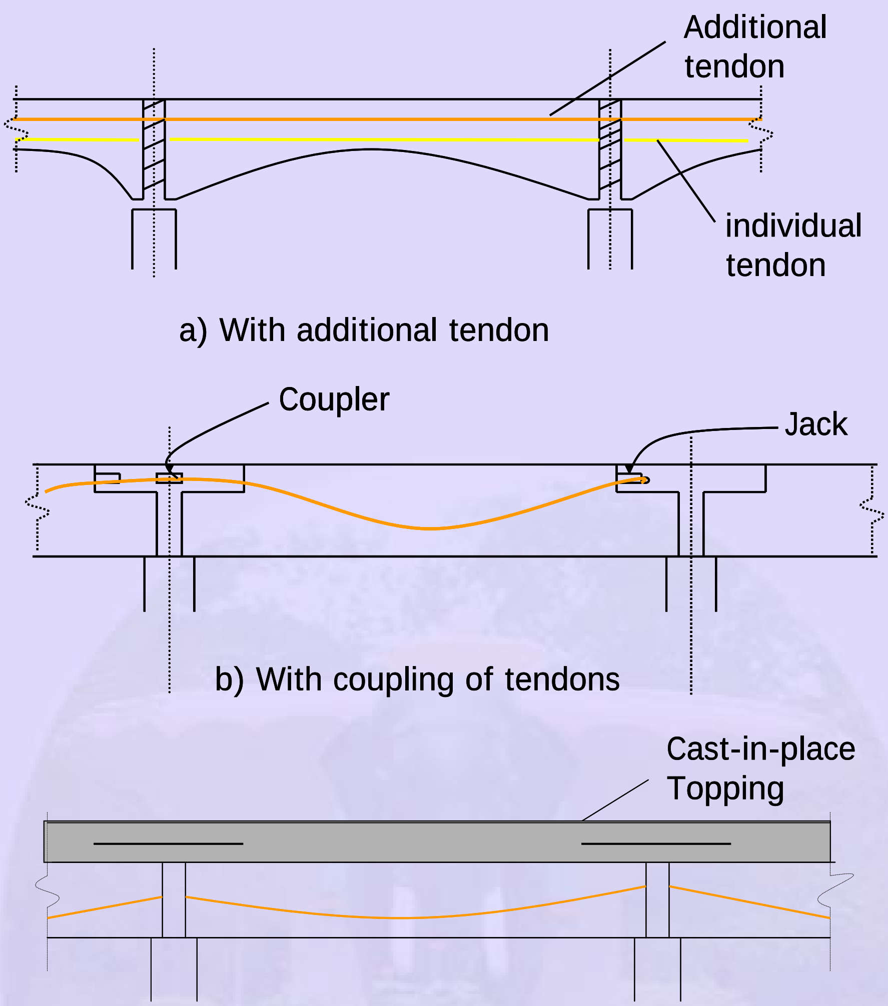
Figure 8-3.5 Profiles of CGS for continuous beams

8.3.4 Partially Continuous Beams

Due to the difficulties in construction of continuous beams, an intermediate system between simply supported beams and continuous beams is adopted. These are called partially continuous beams.

First, the individual precast members are placed at the site. Next continuity is introduced by additional prestressing tendons or coupling the existing tendons. Continuity can also be introduced in a composite construction, where non prestressed continuity reinforcement is introduced in the cast-in-place topping slab.

A few examples are given in the following sketches. Other innovative schemes are also used.



c) Composite construction with continuity reinforcement

Figure 8-3.6 Partially continuous beams

8.3.5 Analysis for Ultimate Strength

The analysis of continuous beams for ultimate strength is difficult for the following reasons.

1) Due to non-linear behaviour, superposition of stresses is not valid.

   ⇒ The concept of load balancing is not truly applicable.

2) The prestressing force varies at the location of cracks.
3) Neglect of the secondary moment due to prestressing is erroneous, unless full moment redistribution is allowed.

Clause 18.6.4 of **IS:1343 - 1980** insists on considering the secondary moment.

8.3.6 Moment Redistribution

It was mentioned in Section 3.4, Analysis of Member under Flexure (Part III), that there is an inconsistency in the traditional analysis at the ultimate state. The demand is calculated based on elastic analysis, whereas the capacity is calculated based on the non-linear limit state analysis. Although the analysis for demand at ultimate is based on an elastic analysis, **IS:1343 - 1980** allows to take advantage of the post-yield deformation of the highly stressed sections in a continuous beam. The underlying concept is known as moment redistribution.

Moment redistribution means the transfer of additional moments to the less stressed sections, as the highly stressed sections with peak moments yield on reaching their ultimate moment capacities.

To apply moment redistribution, the highly stressed sections are designed for lower moments and the less stressed sections are designed to carry higher moments than the values obtained from an elastic analysis. This gives an economical solution.

IS:1343 - 1980, **Clause 21.1.1** specifies the following conditions for moment redistribution.

1) The redistributed moments must be in a state of static equilibrium with the factored external loads.
2) For serviceability requirements, the ultimate moment of resistance at any section (M_{UR}) should not be less than 80% of the moment demand from an elastic analysis (M_u).
3) To limit the demand on post-yield rotation, the reduction in moment at the highly stressed sections is limited to 20% of the numerically largest moment anywhere in the beam calculated by an elastic analysis.
4) To ensure ductile behaviour of the highly stressed sections, the following relationship should be checked.

$$\frac{x_u}{d} + \frac{\delta_M}{100} \leq 0.5 \qquad \textbf{(8-3.5)}$$

Here,

x_u = depth of neutral axis

d = the effective depth

δ_M = the percentage reduction in moment.

Example 8-3.1

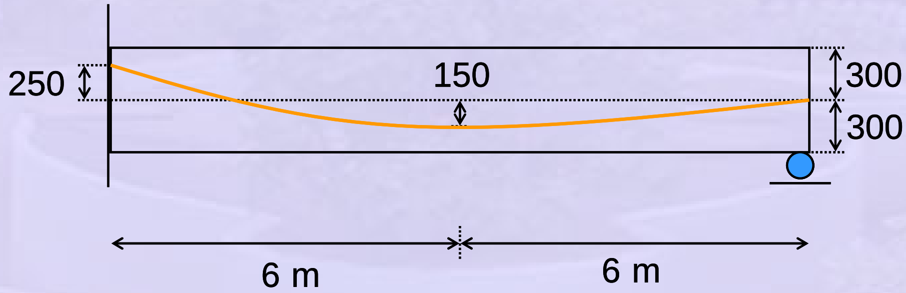
The prestressed concrete beam shown in the figure, is fixed at the left end and roller supported at the right. It is post-tensioned with a single tendon with a parabolic profile, with indicated eccentricities.

a). Locate the pressure line due to application of a prestress force of 1068 kN.

b). Find the primary, secondary and total moments due to prestressing force at the face of the fixed support.

c). What is the magnitude and direction of the reaction produced at the roller by prestressing force?

d). What minor adjustment can be made in the tendon profile to produce a concordant profile?



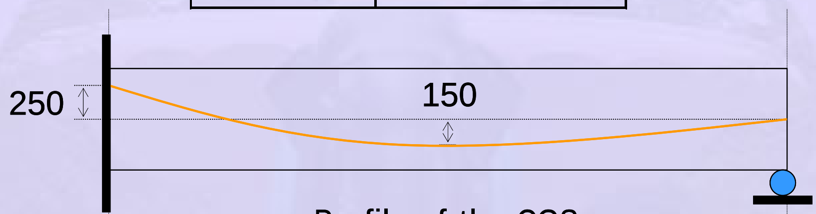
Solution

a) Locate the pressure line.

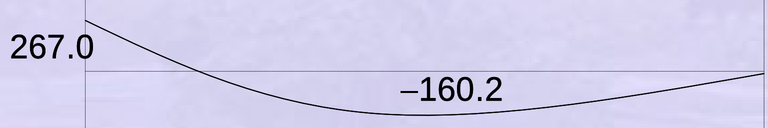
1) Plot M_1 diagram.

The values of M_1 are calculated from $M_1 = P_e e$.

e (m)	M_1 (kN m)
– 0.250	267.0
0.150	– 160.2
0.0	0.0



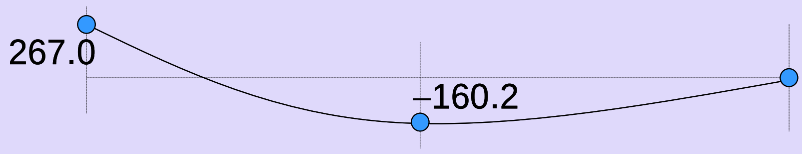
Profile of the CGS



M_1 diagram (kN m)

2) Plot V diagram.

The M_1 diagram is made up of two parabolic segments.

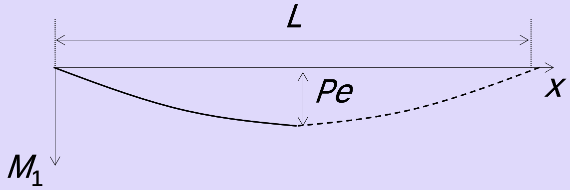


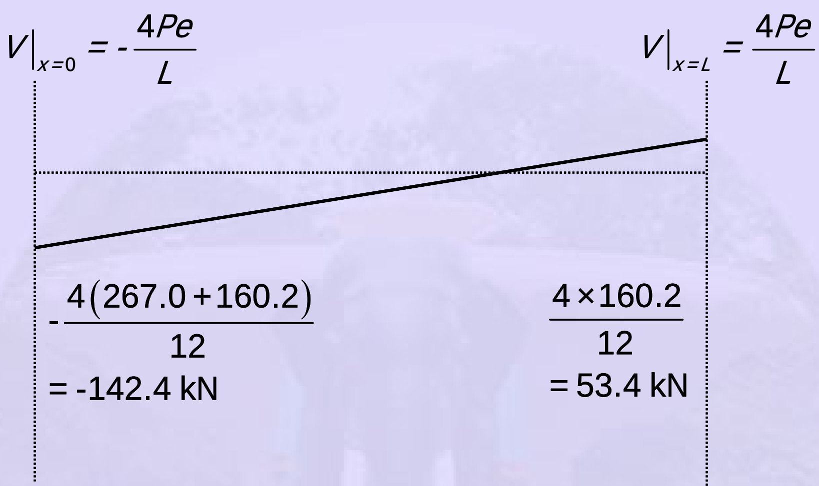
M_1 diagram (kN m)

For each segment,

$$M_1 = -\frac{4Pex}{L^2}(L - x)$$

$$V = \frac{dM_1}{dx}$$
$$= -\frac{4Pe}{L^2}(L - 2x)$$

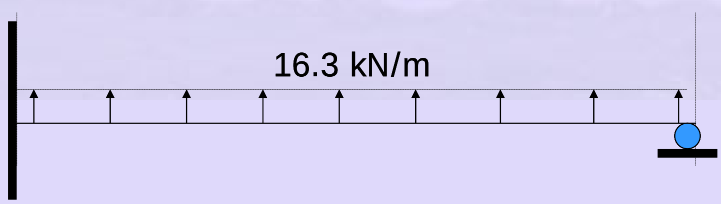




V diagram (kN)

3) Plot equivalent load (w_{eq}) diagram.

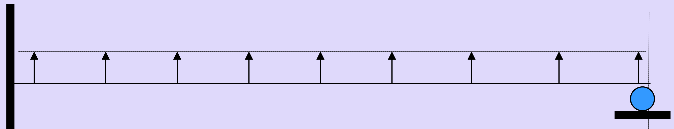
$$w_{eq} = \frac{dV}{dx}$$
$$= \frac{53.4 + 142.4}{12}$$
$$= 16.3 \text{ kN/m}$$



Equivalent load diagram

4) Plot the M_2 diagram.

Calculate moment at supports by moment distribution.



FEM	$\frac{16.3\times12^2}{12}$ $=195.8$	-195.8
Bal		195.8
CO	97.9	
Total	293.7	0

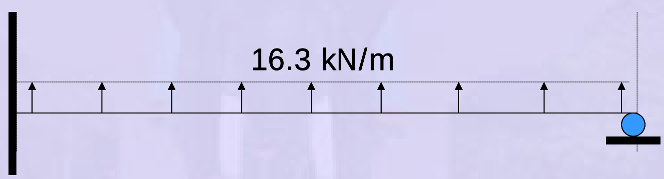
In the previous table,

Bal = Balanced

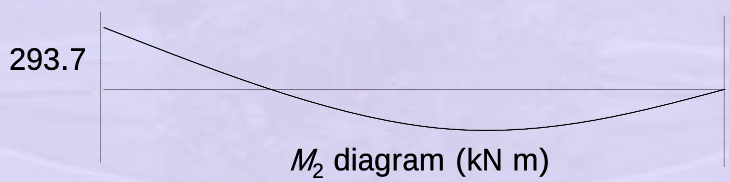
CO = Carry Over moment

FEM = Fixed End Moment.

The moment at the span can be determined from statics. But this is not necessary as will be evident later.



Equivalent load diagram



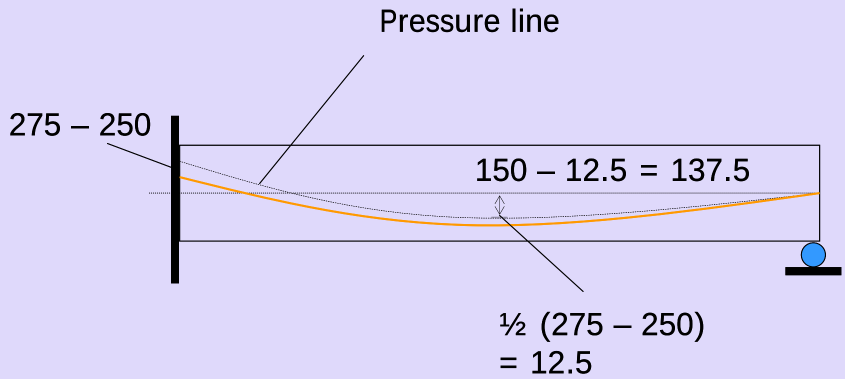
M_2 diagram (kN m)

5) Calculate values of e_c at support.

The values of e_c are calculated from $e_c = M_2/P_e$.

M_2 (kN m)	e_c (m)
293.7	0.275

The deviations of the pressure line from the CGS at the span can be calculated by linear interpolation.



b) Calculation of primary, secondary and total moments.

M_1 = 267.0 kN m primary

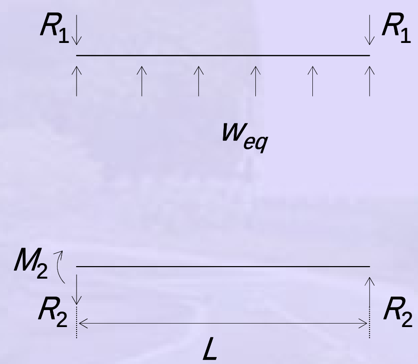
M_2 = 293.7 kN m total

M_1' = $M_2 - M_1$

= 293.7 – 267.0

= 26.7 kN m secondary

c) Calculation of reaction.



$$R_1 = \frac{w_{eq}L}{2}$$

$$= \frac{16.3 \times 12}{2}$$

$$= 97.6 \text{ kN}$$

$$R_2 = \frac{M_2}{L}$$

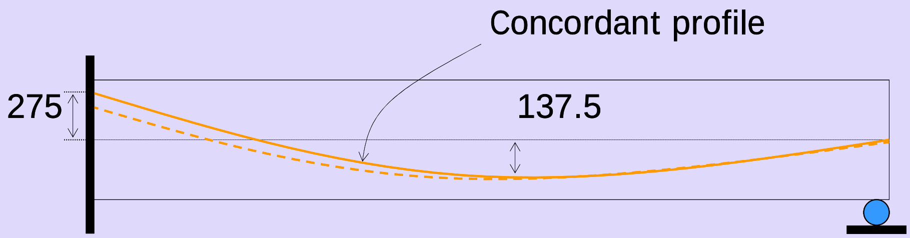
$$= \frac{293.7}{12}$$

$$= 24.5 \text{ kN}$$

$R_1 - R_2$ = 73.1 kN

Resultant reaction at roller is downwards.

d) The tendon can be shifted to coincide with the pressure line to get a concordant profile.



Values in mm.